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# Optimism and pessimism in bargaining and contests☆

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ABSTRACT

This paper uses the rank-dependent expected utility (RDEU) model to capture the effects of *optimism* and *pessimism* on the choice between a pre-trial settlement and a trial (or more generally, between a private settlement and a litigation). These two legal procedures are described as a bargaining game and a contest game, respectively. My models predict that a contest occurs if the aggregate optimism premium in a contest (*AOPC*) is sufficiently high. I also find that the *AOPC* tends to be higher for close cases. Such predictions are consistent with the Priest-Klein empirical observation that a plaintiff's winning probability is often near 50% in many areas of civil litigation. I also show that the highest levels of effort in both a bargaining game and a contest game are exerted when one is moderately optimistic. However, excessive optimism will reduce one's effort level, and hence, one's winning rate. As a result, when faced with an excessively optimistic party, a risk-neutral party may prefer a contest over bargaining.



## 1. Introduction

In a civil dispute, legal decision-makers may choose either to go to trial or to accept a settlement.[1] They may choose a trial over a settlement, even though the monetary gain obtained from a trial could be lower. For example, Kiser et al. (2008) compared rejected settlement offers to actual trial or arbitration verdicts in 2054 cases between 1964 and 2004, and found that the majority of the rejected settlement offers were higher than or equal to the adjudication awards.

In the situation described above, a settlement generates a higher value with certainty, while a trial not only generates a lower expected value but also involves risk and uncertainty. Thus, the choice of a trial over a settlement cannot be explained using an expected utility (EU) model with risk-aversion. However, risk-seeking does not always capture the thinking of a legal decision-maker. For example, an empirical study found that a settlement is more likely to occur when the stakes involved are higher (Fournier and Zuehlke, 1989).[2] With the standard EU model, such a choice contradicts risk-seeking.[3]

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[1] In practice, clients typically make their decisions based primarily on their lawyers' advice, and thus, lawyers and their clients act as a single legal decision-maker (Korobkin and Guthrie, 1997).

[2] There are also empirical findings demonstrating that the American rule leads to a higher trial rate compared to the English rule (Shavell, 1982; Hause, 1989; Hughes and Snyder, 1995). Under the American rule, each party bears their own legal costs, while under the English rule, the loser pays all costs. Thus, for the same case, the stakes would be higher under the English rule than under the American rule.

[3] The proof of this claim is found in Appendix A, Claim 1.

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Priest and Klein (1984) observed an empirical pattern that agrees well with some degree of optimism in legal decision-makers' choice between a trial and a settlement. Their empirical evidence in many areas of civil litigation suggests that the plaintiff's trial winning rate is often around 50%. To explain this, they developed the Priest-Klein trial selection hypothesis (henceforth, the P-K hypothesis):

**The P-K Hypothesis.** When the two parties' gains and losses from a dispute are equal, cases with 50% winning probabilities for each party are more likely to be endogenously selected to go to trial rather than be settled.

To derive this hypothesis, Priest and Klein (1984) propose a trial selection model where both parties make unbiased estimations of their trial-winning probabilities. Such probabilities are exogenous, but are unknown to the parties. The overestimation of one's winning probability is interpreted as optimism (Priest and Klein, 1984). Such interpretation is also adopted in empirical studies (see, e.g. Waldfogel, 1995). In a broader sense, some legal scholars attribute optimism as a hurdle to reaching a settlement (Babcock and Loewenstein, 1997; Korobkin, 2005; Munsinger and Philbin Jr., 2016; O'Grady, 2006). Although the concept of optimism is defined loosely in these papers, it is generally conceived of as a situation where either the perceived or the estimated probability of winning is higher than the true probability.

Motivated by the interest in the effects of optimism – the overestimation or the over-weighting of the probability of winning – on a choice between going to trial or settling, this paper studies a more general situation, where two agents with optimistic or pessimistic attitudes must decide how to allocate a divisible valuable resource. The decision is modeled as a choice between a bargaining game and a winner-take-all contest game. In contrast to the previous literature, this paper does not rely on errors in the assessments of the probability of winning, but emphasizes the notions of optimism and pessimism linked more directly to preferences that differ from EU. Specifically, in the games, the two agents are assumed to have complete information about the strategies and the payoffs. Contest-winning probabilities are objective, thus ambiguity attitudes are irrelevant. Furthermore, the agents are assumed to be risk-neutral in outcome. Each agent's preference is represented by the rank-dependent expected utility (RDEU) model, as in Quiggin (1982, 2012). An optimistic agent will place lower decision weight on the probability of less desirable outcomes and higher decision weight on more desirable outcomes. Meanwhile, a pessimistic agent will do the opposite.[4] Optimism can produce risk-seeking choices even if the agent's utility index for outcome is linear (or concave).[5] Here, in the context of the choice between bargaining and a contest, such preferences can generate the empirical legal patterns mentioned earlier.

The first model in this paper is a simultaneous-move conflict game where the two parties' winning probabilities are exogenous. This situation captures the decision in civil litigation of whether to accept a pre-trial settlement or to go to trial. At this stage, the two parties have already gone through pretrial discovery and conferences, and the case's merits and winning probabilities for each party are clear. Bargaining is conducted under the shadow of a contest, in the sense that a contest occurs if the bargaining breaks down. In this model, the contest that the parties enter if the bargaining breaks down is the same as the contest that the parties voluntarily enter by opting out of bargaining.

In the contest game, each party incurs a contest cost, $C$; while the bargaining game is cost-free. In equilibrium, a contest occurs when the aggregate optimism premium in a contest ($AOPC$) is greater than $2C$. Such a condition can be satisfied even if the bargaining game has a higher expected value after considering the cost of a contest. This prediction matches the empirical pattern described in Kiser et al. (2008). Since the $AOPC$ tends to be higher when the winning probability is closer to 50%, those cases are more likely to go to trials. This situation generates the P-K empirical observation as described in Priest and Klein (1984). Furthermore, one's share from the bargaining game increases (decreases) as one becomes more (less) optimistic relative to her opponent. This suggests that optimism increases one's bargaining power.[6]

This paper also considers a more general situation where the parties' contest game winning probabilities and bargaining game outcomes are endogenously determined by their effort choices in each game. In this setting, the contest that parties voluntarily enter by opting out of bargaining and the contest that they have to enter when bargaining breaks down have different characteristics. This generalization reflects the choice of whether to file litigation in court or to settle in a private venue. A private settlement affects many factors that may change the litigation outcome, including the timing, procedure, and the dynamics between the parties; so the litigation that occurs if settlement fails is different from the litigation that the parties enter without a private settlement. In a bargaining game of this model, the parties will expend their bargaining efforts only; while in a contest game, each party will expend their contest effort in addition to the contest cost, C. The probabilities of winning and losing are determined by the parties' effort choices via a contest success function (CSF).

When the two parties are identical and optimistic, they will always expend the same level of equilibrium effort, and their best response settlement effort choice will be the same as their best response contest effort choice. Therefore, for each party, the expected allocation of the asset will be 50% in a contest, and the share of the settlement will likewise be 50%. Although a contest would be more costly considering the additional contest cost, C, the two parties will choose a contest when $AOPC > 2C$.

When one party is neither optimistic nor pessimistic, and the other party is optimistic, the choice between bargaining and a contest will be more complicated. The model predicts that in both options, the effort from the optimistic party will be higher than the EU prediction only when she is moderately optimistic. An excessively optimistic agent will expend less effort in a contest, and as a result, her contest-winning rate may be low. Indeed, Theorem 4(3) suggests that the optimistic party's highest winning probability in a contest game is only around 0.5124, obtained when she is moderately optimistic ($\alpha \approx 1.3926$); and decreases to 0 as she becomes more optimistic. In contrast, her share from bargaining monotonically increases with her optimism attitudes and is bounded by 0.75. Thus, under some suitable contest cost, a sufficiently optimistic agent may prefer bargaining over a contest. At the same time, her opponent – the neutral party – may have a high winning rate in a contest but can only obtain a low share in bargaining, and thus, the neutral party may prefer a contest over bargaining.

4 This paper takes parties' optimistic and pessimistic attitudes as exogenous and commonly known. To explain and to measure "optimism" are subjects that fall outside the scope of the current paper. However, some research in this direction includes the work of Forgeard and Seligman (2012), who proposed a mechanism whereby an agent chooses options in which she is optimistic due to her biases.

5 For example, both Kahneman and Tversky (1979) and Quiggin (1991) illustrate many hypothetical situations where agents voluntarily insure against loss but still gamble due to such attitudes.

6 The result agrees with the work of Bar-Gill (2005), who showed that optimistic parties can extract better bargaining outcomes.

The rest of this paper is structured as follows. Section 2 summarizes additional related literature. Section 3 introduces the preliminaries. There are two main results in this paper, Theorems 3 and 4. Section 4 discusses the case when the winning probabilities in any contest are exogenous and identical, and develops Theorem 3. Section 5 generalizes the contest game and the bargaining game to the situation where the contest-winning probabilities are endogenously determined by parties' effort choices, and develops Theorem 4. Section 6 discusses and expands Theorem 4. Section 7 concludes. All technical details and proofs are relegated to the appendices.

## 2. Related literature

The P-K hypothesis model in Priest and Klein (1984) assumes that there is an exogenous true case merit for the plaintiff, which is unknown to both the plaintiff and the defendant. Each party makes unbiased predictions of this variable. A trial occurs when the two parties' disagreement of the estimations is greater than a certain threshold that is determined by the costs and the stakes. With the assumption that the prediction errors are independent and identically distributed random variables in a normal distribution, such condition is most likely to be satisfied when a case's merit is close to the legal decision standard. Some authors have applied similar ideas that focus on the parties' beliefs on their winning probabilities to formally prove or to further expand the P-K hypothesis findings, including Lee and Klerman (2016), Hylton and Lin (2011), Shavell (1996), and Gelbach (2018). The overestimation of one's case merit, and therefore the overestimation of one's trial-winning rate, is interpreted as optimism. (see, e.g. Waldfogel, 1995). This paper is similarly motivated by optimism attitudes that may lead to settlement failures. However, optimism and pessimism are modeled as a biased weighting of known objective probabilities, rather than assessment errors of unknown probabilities. Moreover, This paper uses simple two-agent games to provide microfoundations for the P-K hypothesis. This is a different approach from previous literature.

The choice of a contest over bargaining has been studied under some specific settings and with various games for EU-maximizers. For example, when property rights are ambiguously defined, each party must expend effort to support the division rule reached in a settlement. This effort, called the "enforcement cost," is a type of transaction cost. If parties interact over time, they will incur enforcement costs again and again. In comparison, litigation removes the ambiguity and defines the property rights once for all. Thus, parties need not spend any further enforcement costs. Therefore litigation may be more efficient ex-ante due to its precedent-setting power (Robson and Skaperdas, 2008; McBride et al., 2018). Additionally, there are certain other situations where parties may fail to settle due to imperfect information (Bebchuk, 1984; Daughety et al., 1994).

In research on contest games, choices regarding endogenous efforts have been examined from various angles using the EU model. For example, Skaperdas and Gan (1995) found that under EU, different risk attitudes toward an outcome can cause an agent to exert either more or less effort in different settings. Dixit (1987) showed that with strategic precommitments, the efforts expended by both parties will be higher, but the "favorite" will commit more effort than the "underdog." Konrad and Morath (2017) used learning to explain effort dissipation in dynamic multistage contests. Hirshleifer and Osborne (2001) defined a litigation success function – a special contest function that incorporates various aspects into the litigation process – and applied it to examine litigation efforts under various settings of a litigation game. This paper also explore the choice between bargaining and contests, as well as their endogenous effort levels. However, compared to the models mentioned, the models in this paper are much simpler, but the agents here are RDEU-maximizers with linear utility indices, rather than EU-maximizers.[7]

There has been some recent interest in applying RDEU to examine contest models that consider behavior bias. Baharad and Nitzan (2008), for instance, used cumulative prospect theory (CPT) preferences, developed by Tversky and Kahneman (1992), to explore how the number of players and the parameter of distortion affect rent under- and over-dissipation in contests.[8] Keskin (2018) studied CPT in rent-seeking contests with weighting functions and CSF more general than that used in this paper. Although these papers investigate the effects of CPT or RDEU preferences in contests, they focus on the equilibria of general contest games under general CPT or RDEU preferences. In contrast, the emphasis of this paper is on the choice between contests and bargaining, the effort choices in a contest game as well as a bargaining game between two agents, and the application of the predictions to empirical patterns in civil litigation.

## 3. Preliminaries

Adapt the rank-dependent expected utility (RDEU) model for a model of litigation with two parties. Consider a costly winner-take-all contest over a valuable asset, $R$. The contest cost for each party is $C$, and the winner gets the entire asset. There are only two possible outcomes for each agent, winning and losing. Furthermore, the winner's payoff is $y_i^w = R - C$, and the loser's payoff is $y_j^l = -C$, where $w$ denotes winning and $l$ denotes losing, and $i, j = A, B, \quad i \neq j$.

Suppose agent A wins with probability $P$, and loses with probability $1 - P$. The converse applies to agent B. Denote $P_A = (1 - P, P)$, $P_B = (P, 1 - P)$. The RDEU model (Quiggin, 1982, 2012) combines the utility indices for each outcome using decision weights, $h(p)$, where $p$ is the probability measure on the outcomes, and takes values $P_A$ or $P_B$ here.[9] A and B's respective decision weights on the possible outcomes are $h_i(P_i) = \left\{h_i^l(P_i), h_i^w(P_i)\right\}, \quad i = A, B$. This paper considers a special case of RDEU, where the agents are risk-neutral with the utility index

$$u(y) = y. \tag{1}$$

This special case isolates the risk attitudes captured by the nonlinear weighting function $h(p)$, which reflects attitudes toward the probability distribution over outcomes. Denote the risky prospect from contest for $A$ to be $Y_A = (y_A^l, y_A^w; P_A) = (-C, R - C; 1 - P, P)$, and for

[7] In such a case, the RDEU model coincides with the dual theory model developed by Yaari (1987), even though Yaari's axiomatic foundations and assumptions/interpretations are different. In the dual theory model, the utility of a risky prospect is linear in terms of wealth, while the probabilities of different payoff states are not linearly additive. Therefore, perceived risk is processed as attitudes toward probabilities rather than as attitudes towards wealth.

[8] CPT is a more general model compared to RDEU. It captures loss aversion in addition to risk attitudes towards outcomes and towards probabilities, which are likewise captured by RDEU (Tversky and Kahneman, 1992). However, Baharad and Nitzan (2008) did not consider the loss-aversion aspect, and therefore they essentially applied RDEU.

[9] The general RDEU model applies to general outcome spaces and any probability measure, not just the discrete situation examined here (Quiggin, 1982, 2012).

B to be $Y_B = (y_B^l, y_B^w; P_B) = (-C, R-C; P, 1-P)$. RDEU utility for these risky prospects are as follows:

$$\begin{aligned} RDEU(Y_A) &= h_A^l(P_A)(-C) + h_A^w(P_A)(R-C), \\ RDEU(Y_B) &= h_B^l(P_B)(-C) + h_B^w(P_B)(R-C). \end{aligned} \quad (2)$$

To determine each $h_i(P_i), \quad i = A, B$, we first need to rank the outcomes from worst to best by their associated utility index. The order of outcomes in $Y_A$ and $Y_B$ reflects such rankings. Then a transformation function, $q(\cdot)$, is applied to the cumulative probability distribution function (CDF) of the ranked outcomes, where $CDF_j$ indicates the sum of the probabilities for all outcomes ranked below or equal to the $j$th outcome, $CDF_j = \sum_{k=1}^{j} p_k$. The decision weight $h_i^j(p)$ on each outcome $j$ is determined by the difference between $q(CDF_j)$ and $q(CDF_{j-1})$. With the specification that $q(0) = 1, \quad q(1) = 1$, the sum of all decision weights would equal 1. Let A, B's transformation function be the following[10]:

$$q_A(x) = x^{\alpha}, \quad q_B(x) = x^{\beta}. \quad (3)$$

In this model, $\alpha, \beta \in (0, +\infty)$, and these parameters are assumed to be exogenously given and commonly known. With transformation functions (3), optimism and pessimism can be easily defined:

**Definition 1.** With the transformation functions (3), an agent is optimistic if and only if $\alpha > 1$, and is pessimistic if and only if $0 < \alpha < 1$. The same applies to $\beta$. The proof of Definition 1 is in Appendix B.

Apply these notions to $Y_A, Y_B$, we obtain the decision weights for A and B as follows:

$$\begin{aligned} h_A^l(P_A) &= q(1-P) = (1-P)^{\alpha}, \quad h_A^w(P_A) = 1 - q(1-P) = 1 - (1-P)^{\alpha} \\ h_B^l(P_B) &= q(P) = P^{\beta}, \quad h_B^w(P_B) = 1 - q(P) = 1 - P^{\beta} \end{aligned} \quad (4)$$

### 3.1. Optimism in contest game

The contest game considered in this paper is a simple costly winner-take-all contest as described earlier. From Eqs. (2)–(4), the RDEU utilities for A and for B would be:

$$\begin{aligned} \pi_A^c &= RDEU(Y_A) = R(1 - (1-P)^{\alpha}) - C \\ \pi_B^c &= RDEU(Y_B) = R(1 - P^{\beta}) - C \end{aligned} \quad (5)$$

We denote the expected value of the risky prospects $Y_i, i = A, B$ to be $E[Y_i]$, where $E[Y_i] = \sum_{j=l,w} y_i^j P_i^j$; and denote their expected value under $h(P_i)$ to be $RDEU[Y_i] = \sum_{j=l,w} y_i^j h_i^j(P_i)$. With the utility index in (1), $RDEU[Y_i] = RDEU(Y_i)$, and $EU(Y_i) = E[Y_i]$.

Define the optimism premium (OP) for agent $i$ to be the difference between $RDEU[Y_i]$ and $E[Y_i]$.[11] The agent is optimistic if OP is positive, and pessimistic if OP is negative. Applying this definition, we can define the measurements of optimism in a contest, *OPC* and *AOPC*.

**Definition 2** *((OPC and AOPC)).* In the winner-take-all contest game described in this section,

(i) The optimism premiums (OPs) in a contest for parties A and B, denoted as $OPC_A$ and $OPC_B$, respectively, are as follow:

$$\begin{aligned} OPC_A &= RDEU[Y_A] - E[Y_A] = R(1 - (1-P)^{\alpha} - P) \\ OPC_B &= RDEU[Y_B] - E[Y_B] = R((1 - P^{\beta}) - (1-P)) \end{aligned} \quad (7)$$

(ii) The aggregate optimism premium in a contest (AOPC) is defined as the sum of the two parties' OPCs:

$$AOPC = OPC_A + OPC_B = R(1 - (1-P)^{\alpha} - P^{\beta}) \quad (8)$$

$AOPC > 0$ indicates that in the aggregate, the two agents in the contest are optimistic, and thus, the contest can be characterized as an *optimistic contest game*. In the following two lemmas, we list the properties of the *AOPC* in the settings of this paper. The proofs of these two lemmas are found in Appendices B.2 and B.3, respectively. Since Lemmas 1 and 2 will be used to prove one of the main results of this paper, Theorem 3(2), in Section 4, we postpone the discussions of these lemmas until there.

When either $P = 0$ or $P = 1$, by Definition 2, $OPC_A = OPC_B = 0$, and therefore $AOPC = 0$. Thus, without loss of generality, we consider cases where $0 < P < 1$.

[10] Many forms of transformation functions have been applied in RDEU models, including ones with an empirical shape introduced by Tversky and Kahneman (1992).

[11] Quiggin (2012, Chapter 6.4.) provided a general characterization when an agent has any $u(\cdot)$, and when the outcome space is continuous.

$$\begin{aligned} \Delta &= E[y] - CE_{RDEU}[y] \\ &= \left( \int y dF_y - \int y dq(F_y) \right) + \left\{ \int y dq(F_y) - U^{-1} \left( \int U(y) dq(F_y) \right) \right\} \end{aligned} \quad (6)$$

Instead of OP, Quiggin characterized a "pessimism premium", which is the first term in the second line of (6).

**Lemma 1.** *AOPC is affected by A's and B's optimism parameters $\alpha$ and $\beta$ in the following ways:*

(i) *AOPC increases with both $\alpha$ and $\beta$;*
(ii) *If $\alpha \geq 1$ and $\beta \geq 1$, but not $\alpha = \beta = 1$, then AOPC > 0;*
(iii) *When $0 < P < 1$, for any $\beta > 0$, AOPC > 0 if and only if $\alpha > \frac{\log(1-P^{\beta})}{\log(1-P)}$; and for any $\alpha > 0$, AOPC > 0 if and only if $\beta > \frac{\log(1-(1-P)^{\alpha})}{\log P}$.*

Lemma 1(i) suggest that a contest becomes more optimistic (less pessimistic) as the agents become more optimistic (less pessimistic). Definition 1 and Lemma 1(ii) together suggest that the contest is optimistic if at least one agent is optimistic, and the other is not pessimistic. When one party's optimism parameter is given, Lemma 1(iii) describes the required optimism parameter of the other party for a probabilistic contest to be optimistic.

**Lemma 2.** *In the contest game described in this section, for $\alpha$, $\beta \geq 1$, AOPC is continuously differntiable and concave in $P \in [0,1]$, and thus has a maximizer $P^* \in [0,1]$.*

(1) *Let $\alpha = \beta > 1$. AOPC is maximized at $P* = 0.5$.*
(2) *Let $\beta = 1, \alpha > 1$.*
  (i) *AOPC is maximized at $P* = 1 - \alpha^{\frac{1}{1-\alpha}}$. AOPC increases as P increases when $0 \leq P < P^*$, and decreases as P increases when $P* < P \leq 1$;*
  (ii) *$P*$ decreases with $\alpha$. $\lim_{\alpha \to 1, \alpha > 1} P* = 1 - \frac{1}{e}$, $\lim_{\alpha \to \infty} P* = 0$.*

### 3.2. The structure of the bargaining game

In a bargaining game, the two parties divide the asset, R, between themselves. Bargaining costs are normalized to be zero in this model. Let $s$ be a generic notation for the share of an asset $R$ that goes to agent A. Thus, the bargaining outcome for A is $sR$, and for B, it is $(1-s)R$. The feasible set in bargaining, $F$, is all possible divisions of the asset[12]:

$$F = (sR, (1-s)R), \quad s \in [0, 1].$$

This is a convex, compact set. If bargaining breaks down, the two agents engage in a contest. Suppose the winning probabilities in the contest for A and for B are $P$ and $1-P$, respectively. The threat point $d$ is the payoff of the contest when the bargaining breaks down:

$$d = (d_A, d_B) = \left(R(1-(1-P)^{\alpha}) - C, R(1-P^{\beta}) - C\right).$$

Next, we apply the Nash bargaining solution (NBS) to determine the equilibrium on the division of the assets, denoted as $s*$.[13] The NBS maximizes the product of surplus payoffs, $Prod(s)$, defined as:

$$Prod(s) = (sR - R(1-(1-P)^{\alpha}) + C)((1-s)R - (1-P^{\beta})R + C) \tag{9}$$

The NBS $s^*$ exists since the feasible set $F$ is compact, and the objective function $Prod(s)$ in (9) is continuous. It is unique because the objective function $Prod(s)$ is strictly quasi-concave. We use the first order condition to find the unique $s*$ that maximizes $Prod(s)$:

$$\begin{aligned} \frac{dProd(s)}{ds} &= R[(1-s)R - (1-P^{\beta})R + C] - R(sR - R[1-(1-P)^{\alpha}] + C) = 0 \\ \Rightarrow \qquad s* &= \frac{1}{2} + \frac{1}{2}[P^{\beta} - (1-P)^{\alpha}] \end{aligned} \tag{10}$$

The bargaining payoffs in equilibrium are as follows:

$$\begin{aligned} \pi_A^b &= s*R = \left(\frac{1}{2} + \frac{1}{2}[P^{\beta} - (1-P)^{\alpha}]\right) R \\ \pi_B^b &= (1-s*)R = \left(\frac{1}{2} - \frac{1}{2}[P^{\beta} - (1-P)^{\alpha}]\right) R \end{aligned} \tag{11}$$

**Remark 1.** The share and the payoff from bargaining increase as one becomes more optimistic, and they decrease as one's opponent becomes more optimistic.

The proof of this remark is in Appendix B.4. This result suggests that optimism can be considered as a kind of bargaining power.

**Remark 2.** Before proceeding to discuss the models of this paper in Sections 4 and 5, we emphasize that there are two contests, one is the contest that parties enter if bargaining breaks down, and the other is the contest game that parties voluntarily enter. These two contests are the same in the model of Section 4, but they are different in the model of Section 5. More discussions are found in respective sections.

[12] Since $F$ is consist of sure payoffs, the OP is 0 for each party in any feasible type of division. For $A$, there is only one outcome, $sR$, and $P_A = p(sR) = 1$. Thus, by definition, $h(P_A) = q(p(sR)) = 1$. By definition of OP, $OP_A = h(P_A)sR - p(P_A)sR = sR - sR = 0$. Similarly, $OP_B = 0$.

[13] In the bargaining game considered here, it is natural that all the NBS axioms are satisfied. For the axioms, see, e.g., Nash (1950). Thus, NBS is an appropriate solution concept here. NBS is a unique solution for a bargaining game, and can accommodate various forms of the bargaining game, including the formal negotiation game described by Nash in 1953. This negotiation game assumes that agent A threatens agent B by convincing B that if she does not act in compliance with A's demands, then A will carry out a threat; conversely, B does the same toward A (Nash, 1953). Alternatively, the bargaining process can be considered as an infinitely repeating, alternating-offer game with a small breakdown probability. In such a setting, the unique sub-game perfect equilibrium also converges to the NBS (Rubinstein, 1982; Binmore et al., 1986).

| | | B: Bargaining | B: Contest |
|---|---|---|---|
| A | Bargaining | $\pi_A^b, \pi_B^b$ | $\pi_A^c, \pi_B^c$ |
| | Contest | $\pi_A^c, \pi_B^c$ | $\pi_A^c, \pi_B^c$ |

**Fig. 1.** Choice between a contest game and a bargaining game.

## 4. Contest and bargaining games with exogenous probabilities

Consider the decision *between a trial and a pre-trial settlement* during litigation. In this circumstance, the parties would have already gone through pretrial discovery and conferences, and therefore, the efforts would be sunk and the case's merits would be clear. Therefore, a trial that occurs when parties opt out a pre-trial settlement is the same as a trial that occurs when parties' pre-trial settlement fails. In the model, the probabilities in any contest are predetermined, and bargaining does not move the threat point. That is,

**Assumption 1.** The winning probability in any contest is exogenously given to be $P$.

More specifically, A's and B's winning probabilities in both the contest game and the contest which occurs when the bargaining breaks down are exogenously given as $P$ for $A$ and $1-P$ for $B$. In the bargaining game, $A$ and $B$ obtain, respectively, $s*$ and $1-s*$ shares of R with certainty, as determined by the NBS in (10).

As long as one party chooses a contest, bargaining is no longer an option, and the parties will engage in a contest and obtain the contest's payoffs. Thus, the bargaining game is possible only when both parties choose to bargain. A normal form game of this situation is shown in Fig. 1.

It is simple to verify that no matter how $\pi_i^b, \;\; \pi_i^c, \;\; i=A, B$ compare, the contest game is always a Nash equilibrium outcome. The bargaining game is a Nash equilibrium outcome only when $\pi_i^b \geq \pi_i^c$ for both $i=A, B$. To guarantee a unique outcome, I make the following additional assumptions:

**Assumption 2.** For $i=A, B$, if $\pi_i^b \neq \pi_i^c$, $i$ chooses her weakly dominant strategy.

**Assumption 3.** Tie-breaking rule. For $i=A, B$, if $\pi_i^b = \pi_i^c$, $i$ chooses bargaining.

Hence, the bargaining game occurs if and only if:

$$\pi_A^b \geq \pi_A^c \quad \text{and} \quad \pi_B^b \geq \pi_B^c \tag{12}$$

and the contest game occurs if and only if:

$$\pi_A^b < \pi_A^c \quad \text{or} \quad \pi_B^b < \pi_B^c \tag{13}$$

In terms of the AOPC, Theorem 3(1) summarizes the conditions for each game to occur.

**Theorem 3.** *Under Assumptions* 1–3,

(1) *A contest game occurs if and only if*

$$C < \frac{1}{2}AOPC; \tag{14}$$

*and a bargaining game occurs if and only if*

$$C \geq \frac{1}{2}AOPC; \tag{15}$$

(2) *Specifically, the P-K Hypothesis holds in the model of this section.*

The proof for Theorem 3(1) is found in Appendix C. Theorem 3(1) suggests that when P is exogenous, those cases with higher *AOPC*s are more likely to go into trial, while cases with lower *AOPC*s are more likely to settle. Lemmas 1 and 2 are the conditions of when *AOPC* is high. These three results combined suggest possible explanations for the choice of a trial over a settlement.

*Proof of Theorem 3(2).* Lemma 1(i) indicates that the AOPC will become higher as the parties become more optimistic. Thus, with fixed $C$, optimistic parties are more likely to choose a trial over a settlement. This agrees with the explanation of P-K hypothesis in the literature that parties will want to litigate if they are sufficiently optimistic.

Lemma 2(1) suggests that when the parties have similar risk attitudes toward their probabilities ($\alpha = \beta$), then close cases – meaning cases in which the true merit is close to the decision standard, and thus, a plaintiff's winning probability is around 50% – have the highest *AOPC*. Therefore by Theorem 3(1), close cases are more likely to be litigated.

According to Lemma 2(2), when optimistic party A faces neutral party B (i.e., neither optimistic nor pessimistic), AOPC will be maximize at $P^*$, which is less than or equal to a threshold $P_t = 1 - \frac{1}{e} \approx 0.6321$. When $A$'s winning probability is greater than or equal to $P_t$, then the *AOPC* maximizes at $P* = P_t$ for any level of optimism $\alpha > 1$, and decreases as $P$ increases. Thus, for some suitable costs C, for any $\alpha > 1$, it is possible for the contest condition in Theorem 3(1) to be satisfied when $A$'s winning probability is $P_t$, but it will not be satisfied when $A$'s winning probability is higher, for example, $P = 0.9$. Thus, cases where A has a high probability of winning in a contest are more likely to settle compared to cases where $A$ has a winning probability closer to $P_t$. These results, combined with the result found in Theorem 3(1), suggest an interpretation of the P-K hypothesis when $\beta = 1, \alpha > 1$.

Lemma 2(2) further suggests that when the optimistic party A's winning probability is below the threshold $P_t = 1 - \frac{1}{e} \approx 0.6321$, then the AOPC will be highest at $P* = 1 - \alpha^{\frac{1}{1-\alpha}}$. The *AOPC* increases with $P$ with $0 < P < P^*$, and decreases with $P$ when $P^* < P < P_t$. Furthermore, the value of $P^*$ monotonically decreases with $\alpha$. When $\alpha \to 1$, $P*_{\alpha \to 1} \to P_t = 1 - \frac{1}{e} \approx 0.6321$. When $\alpha = 2$, $P*_{\alpha=2} = 0.5$, and when $\alpha = 3$, $P*_{\alpha=3} = 1 - \frac{1}{\sqrt{3}} \approx 0.4223$. When $\alpha$ becomes large, $P*$ approaches 0. That is, when A is very optimistic but her winning probability is less than $P_t = 1 - \frac{1}{e}$, the *AOPC* could be maximized at very small $P^*$. In legal settings, a situation where one party is extremely optimistic ($\alpha \to \infty$) would be rare, and $\alpha$ closer to 1 would be more likely than a large $\alpha$. When $1 < \alpha < 3$, the approximate range of $P*$ is (0.4223, 0.6321). This assumption of a modest level of optimism ($1 < \alpha < 3$) suggests that relatively close cases are, again, more likely to have higher *AOPC*s, and thus, are more likely to be litigated in court. □

Alternatively, we can utilize Lemma 2(2) to explain the rare situations that parties cannot settle even when one party's winning probability at trial is very low. When *A*'s parameter for optimism, $\alpha$, is high, then the *AOPC* may be maximized with a very small contest winning probability for A. For example, when $\alpha = 30$, AOPC is maximized at $P*_{\alpha=30} \approx 0.11$. In such a case, given suitable contest costs, C, a contest may occur when A's winning probability is just 0.11, but not when A's winning probability is higher, say, 0.6. Although the precise prediction of when *AOPC* is maximized relies on the choice of the transformation function $q(\cdot)$, a rough prediction for litigation taking place is that a trial might occur when the excessively optimistic party has a low winning probability under suitable costs. Intuitively, in this situation, A knows for certain that her bargaining payoff is low. When she is sufficiently optimistic, her "risk-seeking" attitude pertaining to her probabilities will lead to a high *AOPC*, and thus, she will prefer to "gamble," and thus, choose the probabilistic contest.

## 5. Contest and bargaining games with endogenous probabilities

This section considers agents' choices between *litigation and a private settlement*, as well as their optimal efforts in each. The private settlement is negotiated under the shadow of litigation, since the parties may still go to court if they are unable to reach an agreement. However, litigation that occurs after a settlement negotiation breaks down is different from litigation that occurs at the outset due to various factors, including prior exerted efforts and resources, timing, and procedural restrictions. Furthermore, the dynamics between the parties might have changed due to their interactions during the settlement. As an example, arbitration is a private settlement process whose outcome may be binding, and such an outcome may only be appealed on very narrow grounds.[14] If, after the arbitration decision, the parties still want to litigate, they will need to go to an appellate court instead of a trial court; and the issues at appeal may be different from the original issues.

If the two parties plan to litigate, they would likely hire trial lawyers and make appropriate investments. On the other hand, if the two parties plan to settle the case in private and avoid litigation, then they would bargain in a private venue. To reflect these considerations in the model, the effort choices in a contest game and in a bargaining game are strategic, and those effort choices of the two parties will determine the outcomes of each game. I use a simple symmetric ratio form of CSF to combine the effort choices to produce the winning probabilities in contests.[15] Let $X_A$ and $X_B$ denote the efforts that *A* and *B* expend, respectively, and let *P* be the probability that *A* wins. Thus,

$$P = \frac{X_A}{X_A + X_B} \tag{17}$$

Fix $X_B$, $P$ increases as $X_A$ increases, and decreases as $X_A$ decreases. The analogous situation applies to *B*.

The timeline of the game in this section is as follows: First, each agent calculates the RDEU payoffs for each game, and based on those payoff calculations, independently chooses to enter the bargaining game or the contest game. When at least one party chooses the contest game, the contest game occurs; and when both parties chooses the bargaining game, the bargaining game occurs. Second, the parties exert corresponding equilibrium efforts for the game that occurs. Third, the parties play the game that they enter, and payoffs are realized. As a result, in a bargaining game, the equilibrium bargaining efforts are exerted in step two before bargaining in step three, and no extra efforts are exerted if bargaining breaks down. We first layout the two games, and then predict agents' choices between the two games.

### *5.1. Contest game with endogenous efforts*

In a contest game, both *A* and *B* choose their own best response effort level (i.e. $X*_A$ or $X*_B$) in order to maximize their own RDEU payoff in a contest. Here the effort cost is a uniform function for both agents:

$$c_i(X) = X, \quad i = A, B$$

[14] Arbitration is one type of alternative dispute resolution (ADR), which is a common form of bargaining in private venues. ADR includes different formats and procedures, for example, arbitration, mediation, and so on. See the section "Dispute Resolution Process" on the American Bar Association website: https://www.americanbar.org/groups/dispute_resolution/resources/DisputeResolutionProcesses/, last accessed August 4, 2020. Also, see the section "ADR Types and Benefits" on the California Courts website: https://www.courts.ca.gov/3074.htm, last accessed August 4, 2020.

[15] A probabilistic CSF is often used to study conflict situations, including rent-seeking, political campaigns, and war, in addition to litigation. In general form, for players $i = 1, 2, \ldots, n$, with the effort vector $X = (X_1, X_2, \ldots, X_n)$, a CSF determines the winning probability $P_i$ for party $i$ as follows.

$$P_i(X) = \frac{f(X_i)}{\sum_{j=1}^{n} f(X_j)} \tag{16}$$

$f(\cdot)$ is the contest technology function; it is a non-negative, strictly increasing function (see, e.g. Jia et al., 2013; Konrad, 2009).

The generic contest payoffs for the parties are:

$$\begin{aligned}\pi_A^c &= R(1-(1-P)^\alpha) - X_A - C = R - R\left(\frac{X_B}{X_A+X_B}\right)^\alpha - X_A - C \\ \pi_B^c &= R(1-P^\beta) - X_B - C = R - R\left(\frac{X_A}{X_A+X_B}\right)^\beta - X_B - C\end{aligned} \tag{18}$$

The optimal levels of effort chosen by each party to maximize the contest RDEU payoffs are the mutual best response efforts, which can be derived by the first order conditions:

$$\begin{aligned}\frac{\partial \pi_A^c}{\partial X_A} = 0 \quad &\Rightarrow R\alpha \frac{X_B^{*\alpha}}{(X*_A + X*_B)^{\alpha+1}} = 1 \\ \frac{\partial \pi_B^c}{\partial X_B} = 0 \quad &\Rightarrow R\beta \frac{X_A^{*\beta}}{(X*_A + X*_B)^{\beta+1}} = 1\end{aligned} \tag{19}$$

The second order conditions for maximization are always satisfied:

$$\frac{\partial^2 \pi_A^c}{\partial X_A^2} = -\alpha(\alpha+1)\left(\frac{X_B^{*\alpha}}{X*_A + X*_B}\right)^{\alpha+2} < 0; \quad \frac{\partial^2 \pi_B^c}{\partial X_B^2} = -\beta(\beta+1)\left(\frac{X_A^{*\beta}}{X*_A + X*_B}\right)^{\beta+2} < 0$$

However, there is no radical solution for $X*_A, X*_B$.[16] This is shown in Lemmas 9 and 10 in Appendix D.1. In Proposition 12 it is shown that when $\alpha, \beta \in \mathbb{N}^+, \alpha > \beta$, with a Mellin transform, we can find the unique real solution in a series with gamma functions:

$$X*_A = R\beta\left[\frac{1}{\alpha}\sum_{r=0}^{\infty}\frac{(-1)^r\Gamma\left(\frac{1+\beta r}{\alpha}\right)}{r!\,\Gamma\left(\frac{1+\beta r}{\alpha}+1-r\right)}\left(\frac{\beta}{\alpha}^{\frac{1}{\alpha}}\right)^r\right]^{\alpha+\alpha\beta},$$

$$X*_B = R\beta\left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha}}\left[\frac{1}{\alpha}\sum_{r=0}^{\infty}\frac{(-1)^r\Gamma\left(\frac{1+\beta r}{\alpha}\right)}{r!\,\Gamma\left(\frac{1+\beta r}{\alpha}+1-r\right)}\left(\frac{\beta}{\alpha}^{\frac{1}{\alpha}}\right)^r\right]^{\alpha\beta+\beta}.$$

Since the general solutions for $X*_A, X*_B$ do not provide any economic intuition, we will leave aside the analytic results and go on to discuss two special cases in Section 6.

### 5.2. Bargaining game with endogenous efforts

In a bargaining game, both parties expend bargaining efforts and negotiate to determine the division of R with the shadow of a contest hanging over them. As explained at the beginning of this section, this contest is different from the voluntary contest game modeled in the previous subsection. If the bargaining breaks down, parties cannot re-optimize and expend the optimal contest efforts. Instead, winning probabilities of this contest are already determined using the equilibrium bargaining efforts expended immediately after bargaining has been decided. The equilibrium bargaining efforts are denoted $X_A^{**}$ and $X_B^{**}$.

A party's bargaining payoff is the share obtained from bargaining subtract their bargaining effort. If $s*$ is the share that goes to A, and if $X_A, X_B$ are the bargaining efforts, then the bargaining payoffs for A and B are

$$\pi_A^b = s^*R - X_A, \pi_B^b = (1-s^*)R - X_B. \tag{20}$$

The bargaining game is solved by backwards induction. There are two stages: exerting efforts, and bargaining. In the bargaining stage, efforts are already exerted and thus need not be considered. Thus, the NBS share $s*$ in bargaining is the same as that found in (10), with probabilities determined by the efforts found in (17):

$$s* = \frac{1}{2} + \frac{1}{2}\left[\left(\frac{X_A}{X_A+X_B}\right)^\beta - \left(\frac{X_B}{X_A+X_B}\right)^\alpha\right] \tag{21}$$

Substituting in (20) with $s*$ as a function of $X_A$ and $X_B$ in (21), and then solving for the mutual best efforts $X_A^{**}, X_B^{**}$ using the first order condition for the system of equations, we reach the following solutions:

$$\begin{aligned}X_A^{**} &= X_B^{**} = \left[\frac{\beta}{2^{\beta+2}} + \frac{\alpha}{2^{\alpha+2}}\right]R, \quad P = \frac{1}{2}, \quad s* = \frac{1}{2} + \frac{1}{2^{\beta+1}} - \frac{1}{2^{\alpha+1}} \\ \pi_A^{b*} &= \left[\frac{1}{2} + \frac{2-\beta}{2^{\beta+2}} - \frac{2+\alpha}{2^{\alpha+2}}\right]R, \quad \pi_B^{b*} = \left[\frac{1}{2} - \frac{2+\beta}{2^{\beta+2}} + \frac{2-\alpha}{2^{\alpha+2}}\right]R\end{aligned} \tag{22}$$

[16] A radical solution is an algebraic equation expressed in terms of the coefficients, using only the operations of addition, multiplication, powers, and radicals.

| | | B | |
|---|---|---|---|
| | | Bargaining | Contest |
| A | Bargaining | $\pi^b_A(X^{**}_A)$,$\pi^b_B(X^{**}_B)$ | $\pi^c_A(X^{*}_A)$,$\pi^c_B(X^{*}_B)$ |
| | Contest | $\pi^c_A(X^{*}_A)$,$\pi^c_B(X^{*}_B)$ | $\pi^c_A(X^{*}_A)$,$\pi^c_B(X^{*}_B)$ |

**Fig. 2.** Choice between a contest game and a bargaining game.

Furthermore, $\alpha, \beta$ must satisfy the second order conditions for maximization. These conditions require:

$$\frac{\alpha(\alpha-3)}{2^{\alpha}} < \frac{\beta(\beta+1)}{2^{\beta}}, \quad \frac{\beta(\beta-3)}{2^{\beta}} < \frac{\alpha(\alpha+1)}{2^{\alpha}} \tag{23}$$

The details of this derivation are found in Appendix D.2. Here, it is shown that although the agents expend the same level of effort, their equilibrium bargaining share differ. If $\alpha > \beta$, then $s* > \frac{1}{2}$. Remark 1, which concerns bargaining power, still applies here: optimism increases one's bargaining payoff.

### 5.3. The choice between a contest game and a bargaining game

In this section, A and B face the choice between a contest game and a bargaining game. They each independently chooses to enter the bargaining game or the contest game based on the payoffs calculated for each game. The bargaining game occurs only when both agents choose bargaining; otherwise the contest game occurs (Fig. 2).

When $\pi^b_i(X^{**}_i) > \pi^c_i(X*_i), \quad i = A, B$, both the bargaining game and the contest game are Nash equilibrium outcomes; while only the bargaining game is the outcome of the weakly dominant strategy equilibrium. As in Section 4, we impose Assumption 2. Therefore a bargaining game occurs in this cases. We also impose the tie breaking rule Assumption 3. Thus, a bargaining game occurs if and only if:

$$\pi^b_A(X^{**}_A, X^{**}_B) \geq \pi^c_A(X*_A, X*_B) \quad \text{and} \quad \pi^b_B(X^{**}_A, X^{**}_B) \geq \pi^c_B(X*_A, X*_B) \tag{24}$$

And a contest game occurs if and only if:

$$\pi^b_A(X^{**}_A, X^{**}_B) < \pi^c_A(X*_A, X*_B) \quad \text{or} \quad \pi^b_B(X^{**}_A, X^{**}_B) < \pi^c_B(X*_A, X*_B) \tag{25}$$

Based on whether bargaining or a contest occurs, the two parties exert their respective best response efforts for that option. Therefore, if a contest occurs, efforts $X*_A$ and $X*_B$ are exerted, and if a bargaining occurs, then efforts $X^{**}_A$ and $X^{**}_B$ are exerted.

To avoid any unnecessary complications, I analyze the following two situations in Section 6.

S1. The two parties have similar risk attitudes towards probabilities of winning, i.e. $\alpha = \beta$.
S2. One party's risk attitude towards their probability of winning is neutral, and the other party is either optimistic or pessimistic, i.e. $\beta = 1$, and $\alpha > 1$ or $\alpha < 1$.

These two situations mainly concern relative measures and are fairly easy to observe. After all, when the model discussed in this paper generates predictions based on the values of $\alpha$ and $\beta$, the discussions are qualitative. The main results of these two special cases are summarized in Theorem 4 below.

**Theorem 4.** *When efforts are chosen endogenously,*

1. *In both special cases S1 and S2,*
   (a) *In both a bargaining game and a contest game, one's equilibrium efforts will be higher than the expected utility (EU) model predictions if and only if her optimism parameter is* $\alpha \in (1, 2)$.
   (b) *One's highest equilibrium effort levels exceed the EU predictions*
      i. *by around* 6% *in both a bargaining game and a contest game in S1;*
      ii. *by around* 3% *in a bargaining game in S2;*
      iii. *and by around* 5% *in a contest game in S2.*
   (c) *One's highest equilibrium effort levels are found*
      i. *when* $\alpha = \frac{1}{\log 2}$ *in both the bargaining game and the contest game in S1, and in the bargaining game in S2;*
      ii. *when* $\alpha \approx 1.3926$ *in the contest game in S2.*
2. *In special case S1,*
   (a) *The best response efforts in a contest game and in a bargaining game are the same:*
   $$X*_A = X*_B = X^{**}_A = X^{**}_B = \frac{\alpha}{2^{\alpha+1}}.$$
   (b) *Theorem* 3 *holds;*
3. *In special case S2,*
   (a) *The optimistic party A's highest contest-winning probability is around* 0.5124, *and is found when* $\alpha$ *is around* 1.3926.
   (b) *For the optimistic party A, the range of share from bargaining is* $(0.5, 0.75)$; *while the range of contest winning probabilities is* $(0, P_{A,\max})$, $P_{A,\max} \approx 0.5124$.
   (c) *With suitable contest cost C, for some* $\alpha \gtrapprox 2.4460$ *dependent on C, the neutral party B may prefer a contest game, while the optimistic party A may prefer a bargaining game.*
   (d) *With suitable contest cost C, when* $\alpha \to \infty$, *both parties prefer a contest game to a bargaining game.*

(e) *When $\alpha > 1$, the neutral party's contest effort will be lower than the EU prediction.*

The proof of Theorem 4 is found in Appendices D.3, E.1, and E.2.3. The discussion for Theorem 4 is in the next section.

## 6. Discussion: choices in games and in efforts with special cases S1 and S2

In this section, for simplicity, $R$ is normalized to be 1. In effect, then, all the other money measures become percentages of R. Since $C$ is a primitive, I use $c$ rather than $C$ to emphasize the fact that $c$ is normalized by $R$, i.e. $c = \frac{C}{R} \geq 0$. In Section 6.1, where the two parties are identical, the contest game and the contest that occurs when bargaining breaks down turn out to have the same endogenous contest winning probabilities; while in Section 6.2, where one party is neutral and the other is optimistic, the two contests generate different contest winning probabilities.

### 6.1. S1: When the two parties have the same attitudes ($\alpha = \beta$)

Since the two parties are identical, their effort levels are identical (see Appendix E.1, proof for Theorem 4(2)(a) for this calculation). I omit subscripts and denote the contest effort as $X*$, and the bargaining effort as $X^{**}$. It is easy to verify that Eq. (23) is satisfied when $\alpha = \beta$. Thus, the parties' equilibrium efforts can be obtained in both bargaining and contest games for all $\alpha > 0$. By CSF in (17), the contest winning probability is $\frac{1}{2}$ in any contest.

**Corollary 5** (Corollary to Theorem 4(2)(a)). *The equilibrium effort level $X* = X^{**}$ strictly increases with $\alpha$ when $0 < \alpha < \frac{1}{\log 2}$, and strictly decreases when $\alpha > \frac{1}{\log 2}$. The maximum level of equilibrium effort is $X*_{\max} = X^{**}_{\max} = \frac{1}{2e\log 2}$, obtained when $\alpha = \frac{1}{\log 2}$. Furthermore,*

1. *The range of equilibrium effort level is $X* = X^{**} \in (0, \frac{1}{2e\log 2})$.*
2. (a) *When $\alpha = 1$ or $\alpha = 2$, then $X* = X^{**} = 0.25$.*
   (b) *When $\alpha \in (1, 2)$, then $X* = X^{**} > 0.25$;*
   (c) *When $\alpha \in (0, 1)$ or $\alpha \in (2, \infty)$, then $X* = X^{**} < 0.25$.*
3. *A's winning probability in the contest game is $P = \frac{1}{2}$, and the A's share in bargaining game is $s* = \frac{1}{2}$.*

The proof of Corollary 5 is found in Appendix E.1. Corollary 5 suggests that, compared to the EU solution, where $\alpha = 1$ and $X* = X^{**} = 0.25$, the best response efforts are lower when the agents are either pessimistic ($\alpha \in (0, 1)$) or overly optimistic ($\alpha \in (2, \infty)$); meanwhile, the best response efforts are higher when the agents are moderately optimistic ($\alpha \in (1, 2)$). Intuitively, a pessimistic party will tend to give up; meanwhile, an excessively optimistic party will severely over-weigh her probability of winning, and thus, expend little effort. Only a moderately optimistic party will be motivated to expend a higher level of effort compared to the baseline EU predictions.

When the efforts are maximized at $\alpha = \frac{1}{\log 2}$, the effort level for each of the two parties will be $X* = X^{**} = \frac{1}{2e\log 2} \approx 0.2654$, which is around 6% higher than the EU model prediction of 0.5. However, the possible reduction of effort level when $\alpha$ has no other restrictions except for $\alpha > 0$, is much more severe: when $\alpha \to 0$ or when $\alpha \to \infty$, the aggregate effort level can be close to 0. For example, when $\alpha < 0.02$ or $\alpha > 9$, then $X* = X^{**} < 0.01$. Thus, this model's predictive power is likely restricted to a limited range of $\alpha$, since, for either a small or large $\alpha$, this model predicts that the agents would expend almost zero effort. In application, there needs to be some realistic restriction on the optimistic or pessimist parameter $\alpha$ for this model to make reasonable qualitative predictions. This justifies for a future research focused on the measurement of optimism and pessimism.

**Corollary 6.** *When the parties' effort levels are chosen endogenously, and when the two RDEU-maximizing parties have the same degree of optimistic or pessimistic attitudes ($\alpha$),*

(1) *Theorem 3 applies.*

(2) *A contest occurs if and only if (i) $0 \leq c < 0.5$, and (ii) $\alpha > \frac{\log\left(\frac{2}{1-2c}\right)}{\log 2}$.*

By Corollary 5, the winning probability for either party in any litigation is always $\frac{1}{2}$, regardless of whether or not parties had attempted to settle. This supports the P-K hypothesis. Therefore the conclusion of Theorem 3(2) holds in the model here. The remaining proof of Corollary 6 is found in Appendix E.1.

From Corollary 6, we can see that when $c = 0$, a contest occurs as long as the two agents are slightly optimistic. The required threshold increases as $c$ increases; and when $c \to 0.5$, a contest will never occur in this model.

Theorem 4(2)(a) and Corollary 6 suggest the following situation: When two RDEU-maximizing parties have the same level of optimism, and the contest cost is relatively low, the two parties will choose a contest. Their probability of winning the contest is 50% each. In the contest game, they expend effort $X*$ and contest cost $c$. However, if the two parties were to bargain, their share would be 50% each as well, and bargaining would only cost them the bargaining effort $X^{**}$ each, where $X* = X^{**}$. Such a choice resembles the empirical pattern found in litigation where parties reject a reasonable settlement offer, even though the outcomes from a trial judgment would be no better than the rejected offer (Kiser et al., 2008).

### 6.2. S2: When one party is neutral ($\beta$=1)

When $\beta = 1$ and $\alpha > 0$, this model generates rich predictions. It is easy to verify that the second order conditions for the bargaining equilibrium in Eq. (23) are satisfied in this situation. We first list Propositions 7, 8 on effort choices in bargaining and in contest, respectively,

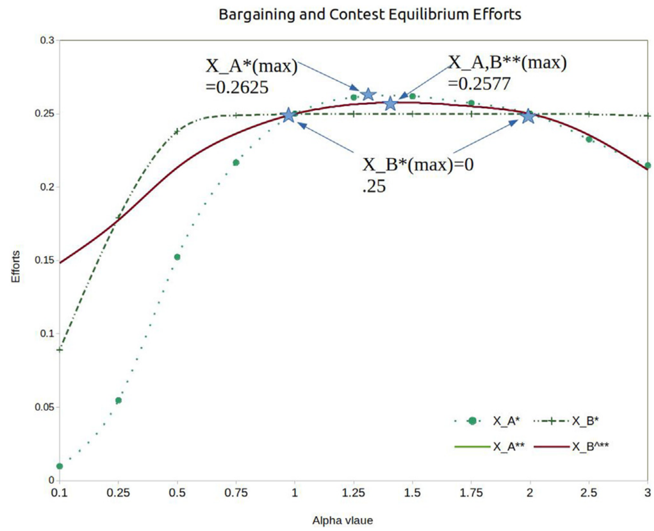


**Fig. 3.** Contest and Bargaining efforts when $\beta = 1$ and $\alpha$ varies.

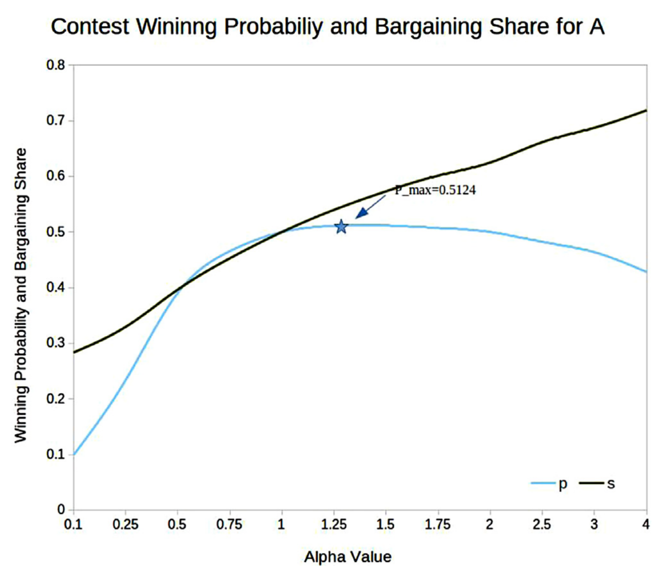


**Fig. 4.** Winning probabilities and bargaining share for A.

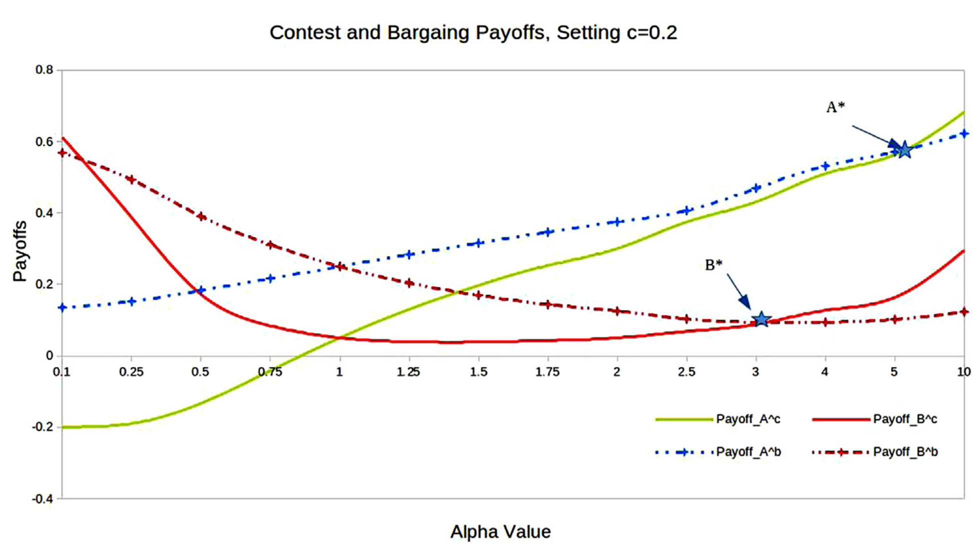


**Fig. 5.** Payoff comparisons of contest and bargaining.

and then illustrate some interesting predictions using Figs. 3–5 based on Propositions 7, 8 and Theorem 4(3).[17] The proofs of these results are in Appendix E.2.

First, notice that according to (22), the bargaining outcomes are:

$$X_A^{**} = X_B^{**} = \frac{1}{8} + \frac{\alpha}{2^{\alpha+2}}, \quad s* = \frac{3}{4} - \frac{1}{2^{\alpha+1}}, \quad \pi_A^{b*} = \frac{5}{8} - \frac{\alpha+2}{2^{\alpha+2}}, \quad \pi_B^{b*} = \frac{1}{8} + \frac{2-\alpha}{2^{\alpha+2}}.$$

**Proposition 7.** *When efforts are chosen endogenously and when one party is neutral ($\beta = 1$), in a bargaining game:*

1. *The equilibrium effort levels are maximized when $\alpha^b_{\max} = \frac{1}{\log 2}$. They increase when $\alpha < \alpha^b_{\max}$, and they decrease when $\alpha > \alpha^b_{\max}$.*
2. *When $\alpha = 1$ or $\alpha = 2$, $X_A^{**} = X_B^{**} = 0.25$; when $\alpha \in (1, 2)$, $X_A^{**} = X_B^{**} > 0.25$; and when $\alpha \in (0, 1) \cup (2, \infty)$, $X_A^{**} = X_B^{**} < 0.25$.*
3. *The range of the best response effort is $0.125 < X^{**} < \frac{2+e\log 2}{8e\log 2}$.*
4. *An optimistic party's ($\alpha > 1$) share from bargaining is $0.5 < s* < 0.75$; while a pessimistic party's ($0 < \alpha < 1$) share from bargaining is $0.25 < s* < 0.5$.*

As equilibrium bargaining efforts of the two parties are equal, we denote it as $X^{**}$. Similar to the situation in Corollary 5(2), $X^{**}$ is greater than the EU model ($\alpha = \beta = 1$) prediction of $X^{**} = 0.25$ only when the agent is moderately optimistic ($1 < \alpha < 2$). Here, the equilibrium bargaining effort level is maximized when $\alpha^b_{\max} = \frac{1}{\log 2}$, and the maximum equilibrium effort expended in bargaining is $X_A^{**} = X_B^{**} = \frac{2+e\log 2}{8e\log 2} \approx 0.2577$. Thus the maximum bargaining equilibrium effort level can exceed the EU prediction of 0.25 by about 3%. However, in contrast to Corollary 5 where effort levels can go to zero, the minimum bargaining efforts here approach $X^{**}_{\min} = 0.125$, as $\alpha \to 0$ or $\alpha \to \infty$.

Lemma 9 in Appendix D.1 implies that contest efforts cannot be expressed in radicals when $\beta = 1$. I describe contest efforts in Proposition 8.

**Proposition 8.** *When efforts are chosen endogenously and when one party is neutral ($\beta = 1$), in a contest game,*

1. *For A, the equilibrium effort level in contest $X*_A$ is maximized at $\alpha^c_{A,\max} \approx 1.3926$. $X*_A$ increases when $\alpha < \alpha^c_{A,\max}$, and decreases when $\alpha > \alpha^c_{A,\max}$.*
2. *For A, $X*_{A,\max} = \left(\frac{\alpha^c_{A,\max}}{e}\right)^2 \approx 0.2625$. $X*_A = 0.25$ when $\alpha = 1, 2$, $X*_A > 0.25$ when $\alpha \in (1, 2)$, and $X*_A < 0.25$ when $\alpha \in (0, 1) \cup (2, +\infty)$. The range of $X*_B$ is $(0, X*_{A,\max})$.*
3. *For B, the equilibrium effort level in contest $X*_B$ is maximized at $\alpha^c_{B,\max} = 1$ or 2, where $X*_{B,\max} = 0.25$. $X*_B < 0.25$ for all $\alpha > 0, \alpha \neq 1, 2$. The range of $X*_B$ is $(0, 0.25)$.*
4. *For B, $X*_B$ increases when $\alpha \in (0, 1) \cup (\alpha^c_{A,\max}, 2)$, and decreases when $\alpha \in (1, \alpha^c_{A,\max}) \cup (2, \infty)$, $\alpha^c_{A,\max} \approx 1.3926$. $X*_B \approx 0.2498$ at $\alpha^c_{A,\max}$.*
5. *$X*_A > X*_B$ when $\alpha \in (1, 2)$, and $X*_B > X*_A$ when $\alpha \in (0, 1) \cup (2, +\infty)$.*

Consistent with the prediction in Corollary 5 and Proposition 7, in a contest, A's equilibrium effort will be higher than the baseline EU model prediction of $X*_A = 0.25$ when she is moderately optimistic ($1 < \alpha < 2$), and will be lower when she is pessimistic or overly optimistic ($\alpha \in (0, 1)$ or $\alpha \in (2, \infty)$). The highest contest equilibrium effort for A is found when $\alpha^c_{A,\max} \approx 1.3926$, and the maximum effort of A is $X*_{A,\max} = \left(\frac{\alpha^c_{A,\max}}{e}\right)^2 \approx 0.2625$, which exceeds the EU prediction by 5%. The neutral party $B$'s effort will always remain lower than the EU model prediction except when $\alpha = 1, 2$, where $X*_B = 0.25$. When $\alpha \in (1, 2)$, then $X*_B \in (X', 0.25)$, $X' \approx 0.2498$.

Fig. 3 illustrates the equilibrium efforts expended by the two parties in bargaining and in contest games for $\alpha \in [0.1, 3]$. On the graph, when $1 < \alpha < 2$, then $X*_A > X^{**}_{A,B} > X*_B$, while the relation between $X*_A$ and $X*_B$ reverses on the range $\alpha \in (0.1, 1) \cup (2, 3)$.

Fig. 4 illustrates the contest winning probabilities and bargaining shares of this setup, which is described in Theorem 4(3)(a)(b). The optimistic party A's winning rate in a contest is maximized when $\alpha^c_{A,\max} \approx 1.3926$. Her maximum winning rate is $P_{\max} = 0.5124$. A's equilibrium share from bargaining, $s*$, increases as $\alpha$ increases. This confirms Remark 1 – that optimism increases bargaining power. As $\alpha \to \infty$, then $s* \to 0.75$. However, when $\alpha \to \infty$, $P \to 0$.

Fig. 4 shows that when $\alpha$ is large, the optimistic party will have a low winning probability in a contest game, but a large share in a bargaining game. Conversely, a neutral party's winning chance in a contest would be high when her opponent is very optimistic, while her settlement share would be low. Indeed, when $\alpha \gtrapprox 2.4460$, then $\pi^c_A(X*_A) - \pi^b_A(X^{**}_A) < \pi_B(X*_B)^c - \pi^b_B(X^{**}_B)$ holds. In other words, for $\alpha \gtrapprox 2.4460$, the gain (loss) from choosing a contest game over a bargaining game will be greater (less) for the neutral party $B$ than for the optimistic party $A$. Theorem 4(3)(c)(d) describes the consequence of this situation.

Specifically, depending on $\alpha$, three cases are possible: (1) both sides of the inequality are greater than $c$, (2) both sides are less than $c$, or (3) the left hand side is less than $c$ while the right hand side is greater than $c$. Hence, given a suitable cost $c \geq 0$, there is some $\alpha \gtrapprox 2.4460$ such that $\pi^c_A(X*_A) - \pi^b_A(X^{**}_A) < c < \pi^c_B(X*_B) - \pi^b_B(X^{**}_B)$ holds. In this case the neutral party B would prefer a contest while the optimistic party A would prefer a settlement.

Fig. 5 illustrates the contest payoff versus settlement payoff when $c = 0.2$ for $\alpha \in [0.1, 10]$. When $\alpha \in [\alpha_{B*}, \alpha_{A*}]$, where $B*, A*$ are the crossings of the contest and settlement payoffs for party B and A, respectively, the optimistic party $A$ prefers bargaining, while the neutral party $B$ prefers a contest. This finding is an interesting result compared to the EU model prediction in the same setting, where a neutral party would never prefer a contest over bargaining.

[17] To draw these figures, I plug in several values of $\alpha$. I get discrete points, and connect them with smooth curves.

## 7. Conclusion

In this paper, I examine litigation patterns in a bargaining game and a contest game in a setting where one of these games is selected endogenously. The models predict that optimistic agent may prefer a risky contest game to a bargaining game, even though the latter has a higher expected value compared to the former. Optimism gives an agent greater bargaining power, and thus would increase that agent's share in bargaining. The models in this paper support P-K hypothesis and explains some other empirical patterns in litigation. Furthermore, the model with endogenous efforts generates rich behavior predictions on the choices between bargaining and contests. For example, in a setting with an optimistic party and a neutral party, the optimistic party's contest winning rate is maximized at around 0.5124 when her optimism parameter is $\alpha \approx 1.3926$, and such winning rate decreases until 0 as $\alpha$ increases. However her share from bargaining increase monotonically. Thus, under suitable costs, an optimistic party may prefer bargaining, while the neutral party may prefer a contest.

Additionally, this paper demonstrated that optimism and pessimism attitudes cause the parties' equilibrium efforts to deviate from EU predictions nonlinearly. In particular, the equilibrium effort findings suggest that with both bargaining and contests, efforts are higher than the EU prediction when the two parties are only moderately optimistic ($\alpha \in (1, 2)$), and are lower than the EU prediction in all other situations. Furthermore, the levels of equilibrium effort decrease quickly when $\alpha > 2$ and when $\alpha < 1$. These results suggest that optimism and pessimism should be taken into consideration when making predictions in games. A natural continuation of this project would be to develop a way to measure the levels of optimism and pessimism for agents in a game.[18]

## Author contributions

**Erya Yang:** Conceptualization, Methodology, Formal analysis, Writing – Original Draft, Writing – Review & Editing.

## Appendix A. Proof for Section 1, Claim 1

**Claim 1.** *In EU model, the pattern that a settlement is easier to be achieved when stakes are higher can only be captured by a concave utility function.*

**Proof.** Let the value in a dispute be $R$, the probability of winning at trial be $p$, and the probability of losing at trial be $1-p$. Let $u(0)=0$. Without considering costs, the risk premium of a trial is the difference between its expected value – that is, the fair settlement payoff – and the certainty equivalence of having a trial (i.e. the sure payoff that generates the same level of EU as a trial). This can be interpreted as the maximum amount that an agent is willing to pay to avoid a trial:

$$RP = EV - CE = pR - u^{-1}(pu(R)).$$

$RP > 0$ when $u(\cdot)$ is concave, and $RP < 0$ when $u(\cdot)$ is convex. That is, a risk-averse agent is willing to pay extra to achieve a fair settlement, and a risk-seeking agent will ask for extra compensation to accept a fair settlement. Taking the derivative with respect to $R$:

$$RP' = p - \frac{pu'(R)}{u'(u^{-1}(pu(R))}$$

Note that $R > u^{-1}(pu(R))$ for any strictly increasing $u(\cdot)$. Therefore, when $u$ is concave, $u'(R) < u'(u^{-1}(pu(R)))$. As a result, $RP' > 0$. The extra amount that a risk-averse agent is willing to pay to avoid a trial increases as $R$ increases. The converse holds for when $u(\cdot)$ is convex: $PR' < 0$. A risk-seeking agent will ask for more compensation to accept a fair settlement as the stakes increase. Therefore, as the stakes increases, it becomes easier for a risk-averse agent to accept a fair settlement, while it becomes more difficult for a risk-seeking agent to do the same. Since a settlement becomes more likely as the stakes increase, at least some of the sampled agents must be risk-averse. □

## Appendix B. Proofs for Section 3. Proofs for Definition 1, Remark 1, Lemmas 1 and 2

*B.1 Proof of Definition 1*

**Proof.** Quiggin (2012) Chapter 6 Lemma 6.1 shows that for any probability measure, an individual is *optimistic* if and only if $q(p) < p$ for all $p \in [0, 1]$, and is *pessimistic* if and only if $q(p) > p$ for all $p \in (0, 1)$:

$$q_i(p) = p^{\gamma} < (>) p \Leftrightarrow \gamma > 1 (< 1).$$

For more general $q(\cdot)$, this implies that there are many situations where an agent is neither optimistic nor pessimistic, i.e. $q(p) \geq p$ for some $p \in [0, 1]$ and $q(p) < p$ for some $p \in [0, 1]$. □

[18] There is literature in the areas of behavioral economics and psychology that evaluates optimism, although the concepts of "optimism" they use are distinct from what I discuss here. For example, Polinsky and Rubinfeld (2002) developed a measurement of optimism using the Survey of Consumer Finance by comparing self-reported life expectancy to statistical tables. Borrowing ideas from their approach, one possibility for measuring RDEU optimism in litigation settings might be to utilize empirical legal research. Relevant works include Kiser et al. (2008), which measured judgment errors in failed settlements.

*B.2 Proof of Lemma 1*

*Proof of Lemma 1*. Without loss of generality, let $R = 1$.

(i) First suppose $0 < P < 1$. $AOPC = 1-(1-P)^{\alpha} - P^{\beta}$. The first order condition with respect to $\alpha$ and $\beta$:

$$\frac{\partial AOPC}{\partial \alpha} = -(1-P)^{\alpha}\log(1-P) > 0$$
$$\frac{\partial AOPC}{\partial \beta} = -P^{\beta}\log P > 0$$

When $P = 1$ or $P = 0$, then $AOPC = 0$ for any $\alpha$, $\beta$. Therefore, $AOPC$ always increases with $\alpha$ and with $\beta$.

(ii) Suppose $\alpha = 1$. Then, $AOPC = 1-(1-P) - P^{\beta} = P - P^{\beta} > 0$ if $\beta > 1$. By (i), $AOPC$ increases with $\alpha$. Therefore, for all $\alpha \geq 1$, $\beta > 1$, $AOPC > 0$. Similarly, suppose $\beta = 1$. Thus, $AOPC = 1-(1-P)^{\alpha} - P > 0$ if $\alpha > 1$. By (i), $AOPC$ increases with $\beta$. Therefore for all $\beta \geq 1$, $\alpha > 1$, $AOPC > 0$.

(iii) Suppose $AOPC > 0$. Then,

$$\begin{aligned} AOPC &= 1-(1-P)^{\alpha} - P^{\beta} > 0 \Leftrightarrow (1-P^{\beta}) > (1-P)^{\alpha} \\ &\Leftrightarrow \log(1-P^{\beta}) > \alpha\log(1-P) \Leftrightarrow \alpha > \frac{\log(1-P^{\beta})}{\log(1-P)}; \\ AOPC &= 1-(1-P)^{\alpha} - P^{\beta} > 0 \Leftrightarrow 1-(1-P)^{\alpha} > P^{\beta} \\ &\Leftrightarrow \log(1-(1-P)^{\alpha}) > \beta\log P \Leftrightarrow \beta > \frac{\log(1-(1-P)^{\alpha})}{\log P} \end{aligned}$$

□

*B.3 Proof of Lemma 2*

**Proof.** It is easy to see that $AOPC$ as defined in (10) is continuously differentiable for any $\alpha, \beta \geq 1$. Its second order derivative is negative and thus it is concave.

1. The first order condition with respect to $P$ is

$$\frac{\partial AOPC}{\partial P} = R\left(\alpha(1-P)^{\alpha-1} - \beta P^{\beta-1}\right)$$

First, if $\beta = \alpha > 1$, then

$$\frac{\partial AOPC}{\partial P} = \alpha R P^{\alpha-1}\left(\left(\frac{1-P}{P}\right)^{\alpha-1} - 1\right)$$

The partial derivative is greater than 0 when $0 < P < 0.5$; less than 0 when $0.5 < P < 1$; and equal to 0 when P=0.5. Therefore, $AOPC$ is maximized at $P = 0.5$.

2. Consider the case where $\alpha > 1$, $\beta = 1$. Denote

$$f(P,\alpha) = \frac{1}{R}\frac{\partial AOPC}{\partial P} = \alpha(1-P)^{\alpha-1} - 1$$

Because
$f(P,\alpha) = \alpha(1-P*)^{\alpha-1} - 1 = 0 \Leftrightarrow P* = 1-\alpha^{\frac{1}{1-\alpha}}$,
and
$f(P,\alpha) < (>) 0 \quad when \quad P > (<) P*, 0 \leq P \leq 1$
$$\frac{\partial^2 AOPC}{\partial P^2} = \frac{\partial f(P,\alpha)}{\partial P} = -\alpha(\alpha-1)(1-P)^{\alpha-2} < 0,$$
$AOPC$ is maximized at $P* = 1-\alpha^{\frac{1}{1-\alpha}} \in (0, 1-\frac{1}{e})$, and it is also concave. Next, when $\alpha > 1$,
$$\frac{\partial P*}{\partial \alpha} = \frac{\alpha^{-\alpha/(\alpha-1)}}{(\alpha-1)^2}(\alpha - 1 - \alpha\log\alpha) < 0$$
Thus, $P*$ decreases as $\alpha$ increases, and $\lim_{\alpha\to 1, \alpha>1} P* = 1-\frac{1}{e}$, and $\lim_{\alpha\to\infty} P* = 0$.

□

*B.4 Proof of Remark 1*

**Proof.** From (10),

$$s* = \frac{1}{2} + \frac{1}{2}\left[P^{\beta} - (1-P)^{\alpha}\right]$$

$$\frac{\partial s*}{\partial \alpha} = -0.5\log(1-p)(1-p)^{\alpha} > 0, \quad \frac{\partial s*}{\partial \beta} = 0.5p^{\beta}\log p < 0 \tag{26}$$

Thus A's equilibrium share $s*$ increases as $\alpha$ increases; and decreases as $\beta$ increases. The opposite applies to B. Therefore, a party's share and their payoff from bargaining increase with their own optimism level and decrease with the rise of the other party's optimism level. □

## Appendix C. Proof for Section 4: proof of Theorem 3(1)

Theorem 3(1), which is the contest condition for a contest to occur, is proved here. Theorem 3(2) is proved in the main text.

*Proof of Theorem 3(1).* As RDEU-maximizers, both $A$ and $B$ compare their respective payoffs in a contest game ($\pi^c$), as determined in (5) and in a bargaining game ($\pi^b$), as determined in (11), in order to choose the option with the higher RDEU utility:

$$\begin{aligned}
\pi^c_A > \pi^b_A \quad &\Leftrightarrow R(1-(1-P)^{\alpha}) - C > (\frac{1}{2} + \frac{1}{2}[P^{\beta} - (1-P)^{\alpha}])R \\
&\Leftrightarrow \quad C < \frac{1}{2}[1-(1-P)^{\alpha} - P^{\beta}]R \\
\pi^c_B > \pi^b_B \quad &\Leftrightarrow R(1-P^{\beta}) - C > (\frac{1}{2} - \frac{1}{2}[P^{\beta} - (1-P)^{\alpha}])R \\
&\Leftrightarrow \quad C < \frac{1}{2}[1-(1-P)^{\alpha} - P^{\beta}]R
\end{aligned}$$

Therefore, $A$ and $B$ have the same decision criteria, and the right-hand side is $\frac{1}{2}$ of the AOPC as defined in Eqn 8. □

## Appendix D. Calculations and proofs for Section 5

*D.1 Section 5.1, contest game*

WLOG let $R = 1$. Then all the monetary terms are percentages of R. Thus, the first order conditions in (19) become the following:

$$\frac{X*_B}{X*_A + X*_B} = \left(\frac{X*_A + X*_B}{\alpha}\right)^{\frac{1}{\alpha}}, \quad \frac{X*_A}{X*_A + X*_B} = \left(\frac{X*_A + X*_B}{\beta}\right)^{\frac{1}{\beta}}$$

WLOG let $\alpha \geq \beta$, and denote $y = \left(\frac{X*_A + X*_B}{\alpha}\right)^{\frac{1}{\alpha}}$. Adding the two equations above, it follows

$$1 = y + \left(\frac{\alpha}{\beta}y^{\alpha}\right)^{\frac{1}{\beta}} = y + \left(\frac{\alpha}{\beta}\right)^{\frac{1}{\beta}} y^{\frac{\alpha}{\beta}}$$

Thus, if one can find zeros, i.e., roots, of the function $f(y) = y + \left(\frac{\alpha}{\beta}\right)^{\frac{1}{\beta}} y^{\frac{\alpha}{\beta}} - 1 = 0$, then one can locate $X*_A, X*_B$ using $y$:

$$X*_A = \alpha\left(\frac{\alpha}{\beta}\right)^{\frac{1}{\beta}} y^{\alpha + \frac{\alpha}{\beta}} = \alpha y^{\alpha}(1-y); \quad X*_B = \alpha y^{\alpha+1} \tag{27}$$

**Lemma 9.** *The roots of polynomial $f(y) = 5y^5 + y - 1$ cannot be expressed in radicals.*

**Proof.** By the Abel-Ruffini theorem, some polynomial equations with a degree greater than or equal to 5 do not have radical solutions. In fact, an irreducible polynomial $f(x)$ in $\mathbb{Q}[x]$ of degree 5 can be solved by radicals if and only if its Galois group is contained in the Frobenius group $\mathbb{F}_{20}$ of order 20. Dummit, 1991 shows that this is the case if and only if the corresponding resolvent sextic $f_{20}(x)$ has a rational root. If the quintic is in the form of $f(x) = x^5 + Ax + B$, $A, B \in \mathbb{Q}$, then the polynomial $f_{20}(x)$ would be:
$f_{20}(x) = x^6 + 8Ax^5 + 40A^2x^4 + 160A^3x^3 + 400A^4x^2 + (512A^5 - 3125B^4)x - 9375AB^4 + 256A^6$.

First, we show that $f(y) = 5y^5 + y - 1$ is irreducible. By Gauss's lemma of irreducibly, if the coefficients of a non-constant polynomial $f(y)$ are relatively prime and $f(y)$ is irreducible in $\mathbb{Z}[y]$, then $f$ is irreducible in $\mathbb{Q}[y]$.

By the property of $\mathbb{Z}[y]$ – that for every prime $p$, there is a reduction map from $\mathbb{Z}[y]$ to $\mathbb{F}_p[y]$ – if $f$ is irreducible over $\mathbb{F}_p[y]$ then $f$ is irreducible in $\mathbb{Z}[y]$. Take $\mathbb{F}_3 = \{0, 1, 2\}$. Then, we only need to check whether the mod-3-polynomial $y^5 + 2y + 1$ is irreducible. Take the complete system of residuals 0,1,2, we see that $y^5 + 2y + 1$ (mod 3) has no root in them. Therefore it has no linear factor. The only irreducible monic quadratics in $\mathbb{F}_3$ are $x^2 + 1$, $x^2 + x + 2$, and $x^2 + 2x + 2$; and none of them is a factor of $y^5 + 2y + 1$ , checking by Euclidean algorithm. Thus, $f(y)$ is irreducible over $\mathbb{F}_3[y]$. Therefore, $f(y)$ is irreducible in $\mathbb{Z}[y]$, and is irreducible in $\mathbb{Q}[y]$.

Next, we find the corresponding resolvent polynomial of degree 6 $f_{20}(y)$ for $\frac{f(y)}{5} = y^5 + \frac{1}{5}y - \frac{1}{5}$, where $A = \frac{1}{5}, \quad B = -\frac{1}{5}$. We denote it by $g(y)$:

$$g(y) = \frac{1}{15,625}(15,625y^6 + 25,000y^5 + 25,000y^4 + 20,000y^3 + 10,000y^2 - 75,565y - 46,619)$$

By the rational root theorem, if there is a rational root, it must be of the form of $\pm\frac{a}{b}$, where $a$ is a factor of 15, 625, and $b$ is a factor of 46, 619. Since 46, 619 is a prime number, and because $15,625 = 5^6$, the possible rational roots are:

$$\pm\frac{1}{46,619}, \pm\frac{5}{46,619}, \pm\frac{25}{46,619}, \pm\frac{125}{46,619}, \pm\frac{625}{46,619}, \pm\frac{3125}{46,619}, \pm\frac{15,625}{46,619},$$

$$\pm 1, \pm 5, \pm 25, \pm 125, \pm 625, \pm 3125, \pm 15,625.$$

However, none of the above is a root for $g(y)$. Thus, $g(y)$ has no rational root; hence, the irreducible polynomial $f(y) = 5y^5 + y - 1$ has no root in radicals. □

**Lemma 10.** *The roots of the function* $f(y) = y + \left(\frac{\alpha}{\beta}\right)^{\frac{1}{\beta}} y^{\frac{\alpha}{\beta}} - 1$ *cannot be expressed using general* $\alpha$ *and* $\beta$ *in radicals.*

**Proof.** If $f(y)$ has roots in radicals, then the special case where $\alpha = 5$ and $\beta = 1$ has radical roots. We have demonstrated that this is not the case in Lemma 9, i.e., the polynomial equation $5y^5 + y - 1 = 0$ does not have radical roots. Thus $f(y) = y + \frac{\alpha}{\beta}^{\frac{1}{\beta}} y^{\frac{\alpha}{\beta}} - 1$ has no radical solution. □

Nevertheless, even though we cannot express the solution for $y$ in radicals, $f(y)$ still has a solution in real numbers, $\mathbb{R}$.

**Lemma 11.** *The polynomial* $f(y) = 5y^5 + y - 1$ *has one real root.*

**Proof.** This can be shown using Sturm's theorem. First we construct a Sturm chain, such that

$$P_0 = P, \quad P_1 = P', \quad P_{i+1} = -rem(P_{i-1}, P_i).$$

Here,

$$P_0 = 5y^5 + y - 1, \quad P_1 = 25y^4 + 1, \quad P_2 = -\frac{4}{5}y + 1, \quad P_3 = -\frac{15,881}{256}.$$

Since $y = \frac{X*_B}{X*_A + X*_B}$, $X*_A, X*_B > 0$, then $y \in [0, 1]$. On the domain [0, 1], at 0, the signs of $P_0$ to $P_3$ are $(-, +, +, -)$; and at 1, the signs are $(+, +, +, -)$. Thus, $V(0) = 2$, $V(1) = 1$. Since $V(0) - V(1) = 1$, there is exactly one real root in [0, 1]. □In fact, a more general result holds.

**Proposition 12.** *For* $\alpha > \beta, \alpha, \beta \in \mathbb{N}^+$, *there is exactly one general solution in* $\mathbb{R}$ *for* $y + \frac{\alpha}{\beta}^{\frac{1}{\beta}} y^{\frac{\alpha}{\beta}} = 1$, *which is a solution in a series:*

$$y = \sum_{r=0}^{\infty} \frac{\Gamma\left(1 + r\frac{\alpha}{\beta}\right)}{\Gamma\left(2 + r\frac{\alpha}{\beta} - r\right) r!} (-1)^r \left(\frac{\alpha}{\beta}\right)^{\frac{r}{\beta}}$$

**Proof.** In Hochstadt (1986, Chapter 3.8), using Mellin transform, the author proved that the trinomial $y^n + ay^p - 1 = 0, \quad n > p$ equation has the following series solutions:

$$y(a) = \frac{1}{n} \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma\left(\frac{1+pr}{n}\right) a^r}{r! \Gamma\left(\frac{1+pr}{n} + 1 - r\right)}.$$

Let $n = \alpha, \quad p = \beta$, and $z = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{\alpha\beta}} y^{\frac{1}{\beta}}$. From the given function $y + \frac{\alpha}{\beta}^{\frac{1}{\beta}} y^{\frac{\alpha}{\beta}} = 1$ we obtain the polynomial of $z$:

$$z^\alpha + \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha}} z^\beta - 1 = 0.$$

Therefore, the solution in the series of this trinomial is:

$$z\left(\frac{\beta}{\alpha}^{\frac{1}{\alpha}}\right) = \frac{1}{\alpha} \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma(\frac{1+\beta r}{\alpha})}{r! \Gamma(\frac{1+\beta r}{\alpha} + 1 - r)} \left(\frac{\beta}{\alpha}^{\frac{1}{\alpha}}\right)^r$$

By the fundamental theorem of algebra, there are $n$ roots in total for this trinomial; and the remaining $n-1$ solutions are found by evaluating $\varepsilon^k z(\varepsilon^{\beta k}(\frac{\beta}{\alpha})^{\frac{1}{\alpha}})$, where $\varepsilon = e^{i\frac{2\pi}{n}}$, $k = 1, 2, \ldots, n-1$, which are complex rotations of $z$. Since $y = \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha}} z^{\beta}$, we have the following real solution for $y$:

$$y = \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha}} \left[\frac{1}{\alpha}\sum_{r=0}^{\infty} \frac{(-1)^r \Gamma\left(\frac{1+\beta r}{\alpha}\right)}{r!\Gamma\left(\frac{1+\beta r}{\alpha}+1-r\right)} \left(\frac{\beta}{\alpha}^{\frac{1}{\alpha}}\right)^r\right]^{\beta}$$

Substitute the solution for $y$ into (27) and multiply $R$, we obtain:

$$\begin{aligned}
X*_A &= R\alpha\left(\frac{\alpha}{\beta}\right)^{\frac{1}{\beta}} \left(\left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha}} \left[\frac{1}{\alpha}\sum_{r=0}^{\infty} \frac{(-1)^r \Gamma\left(\frac{1+\beta r}{\alpha}\right)}{r!\Gamma\left(\frac{1+\beta r}{\alpha}+1-r\right)} \left(\frac{\beta}{\alpha}^{\frac{1}{\alpha}}\right)^r\right]^{\beta}\right)^{\alpha+\frac{\alpha}{\beta}} \\
&= R\beta\left[\frac{1}{\alpha}\sum_{r=0}^{\infty} \frac{(-1)^r \Gamma\left(\frac{1+\beta r}{\alpha}\right)}{r!\Gamma\left(\frac{1+\beta r}{\alpha}+1-r\right)} \left(\frac{\beta}{\alpha}^{\frac{1}{\alpha}}\right)^r\right]^{\alpha+\alpha\beta} \\
X*_B &= R\alpha\left(\left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha}} \left[\frac{1}{\alpha}\sum_{r=0}^{\infty} \frac{(-1)^r \Gamma\left(\frac{1+\beta r}{\alpha}\right)}{r!\Gamma\left(\frac{1+\beta r}{\alpha}+1-r\right)} \left(\frac{\beta}{\alpha}^{\frac{1}{\alpha}}\right)^r\right]^{\beta}\right)^{\alpha+1} \\
&= R\beta\left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha}} \left[\frac{1}{\alpha}\sum_{r=0}^{\infty} \frac{(-1)^r \Gamma\left(\frac{1+\beta r}{\alpha}\right)}{r!\Gamma\left(\frac{1+\beta r}{\alpha}+1-r\right)} \left(\frac{\beta}{\alpha}^{\frac{1}{\alpha}}\right)^r\right]^{\alpha\beta+\beta}
\end{aligned} \tag{28}$$

□

*D.2 Section 5.2, bargaining with endogenous efforts*

We solve the game by backwards induction. The bargaining share is first determined as in (21). Thus, the parties choose efforts to maximized the following bargaining payoffs:

$$\pi_A^b = \left(\frac{1}{2} + \frac{1}{2}\left[\left(\frac{X_A}{X_A+X_B}\right)^{\beta} - \left(\frac{X_B}{X_A+X_B}\right)^{\alpha}\right]\right)R - X_A$$

$$\pi_B^b = \left(\frac{1}{2} - \frac{1}{2}\left[\left(\frac{X_A}{X_A+X_B}\right)^{\beta} - \left(\frac{X_B}{X_A+X_B}\right)^{\alpha}\right]\right)R - X_B$$

The mutual best response efforts that maximize the settlements payoffs is obtained by the first order condition:

$$\frac{\partial \pi_A^b}{\partial X_A} = 0 \Rightarrow \frac{1}{2}\beta R\frac{X_A^{**\beta-1}X_B^{**}}{(X_A^{**}+X_B^{**})^{\beta+1}} + \frac{1}{2}\alpha R\frac{X_B^{**\alpha}}{(X_A^{**}+X_B^{**})^{\alpha+1}} = 1$$

$$\frac{\partial \pi_B^b}{\partial X_B} = 0 \Rightarrow \frac{1}{2}\beta R\frac{X_A^{**\beta}}{(X_A^{**}+X_B^{**})^{\beta+1}} + \frac{1}{2}\alpha R\frac{X_B^{**\alpha-1}X_A^{**}}{(X_A^{**}+X_B^{**})^{\alpha+1}} = 1$$

Multiply the equation after the arrow on the first line by $\frac{X_A^{**}}{X_B^{**}}$,

$$\frac{1}{2}\beta R\frac{X_A^{**\beta-1}X_B^{**}}{(X_A^{**}+X_B^{**})^{\beta+1}} \times \frac{X_A^{**}}{X_B^{**}} + \frac{1}{2}\alpha R\frac{X_B^{**\alpha}}{(X_A^{**}+X_B^{**})^{\alpha+1}} \times \frac{X_A^{**}}{X_B^{**}} = \frac{X_A^{**}}{X_B^{**}}$$

The left hand side simplifies to be the same as the left hand side of the equation after the arrow on the second line. Thus, $X_A^{**} = X_B^{**}$. By this we can easily calculate $X_A^{**}$ and $X_B^{**}$ from either of the two the equations after the arrow. By (17) we obtain $P$, and by (21) we obtain $s*$. Plug these to (20) we obtain $\pi_A^b(X^{**}{}_A)$ and $\pi_B^b(X^{**}{}_B)$.

Thus,

$$X_A^{**} = X_B^{**} = \left[\frac{\beta}{2^{\beta+2}} + \frac{\alpha}{2^{\alpha+2}}\right]R, \quad P = \frac{1}{2}, \quad s* = \frac{1}{2} + \frac{1}{2^{\beta+1}} - \frac{1}{2^{\alpha+1}},$$
$$\pi_A^b\left(X_A^{**}\right) = \left[\frac{1}{2} + \frac{2-\beta}{2^{\beta+2}} - \frac{2+\alpha}{2^{\alpha+2}}\right]R, \quad \pi_B^b\left(X_B^{**}\right) = \left[\frac{1}{2} - \frac{2+\beta}{2^{\beta+2}} + \frac{2-\alpha}{2^{\alpha+2}}\right]R \tag{29}$$

Additionally, the second order condition for maximization must also be satisfied:

$$\frac{\partial^2 \pi_A^b}{\partial X_A^2} = \frac{1}{2}\beta R \frac{X_A^{\beta-2}X_B}{(X_A+X_B)^{\beta+2}}\left[(\beta-1)X_B - 2X_A\right] - \frac{1}{2}\alpha R \frac{(\alpha+1)X_B^{\alpha}}{(X_A+X_B)^{\alpha+2}} < 0$$
$$\frac{\partial^2 \pi_B^b}{\partial X_B^2} = \frac{1}{2}\alpha R \frac{X_B^{\alpha-2}X_A}{(X_A+X_B)^{\alpha+2}}\left[(\alpha-1)X_A - 2X_B\right] - \frac{1}{2}\beta R \frac{(\beta+1)X_A^{\beta}}{(X_A+X_B)^{\beta+2}} < 0$$

At $X_A^{**}, X_B^{**}$, the second order condition is satisfied when the following inequality of $\alpha, \beta$ holds:

$$\frac{\alpha(\alpha-3)}{2^{\alpha}} < \frac{\beta(\beta+1)}{2^{\beta}}, \quad \frac{\beta(\beta-3)}{2^{\beta}} < \frac{\alpha(\alpha+1)}{2^{\alpha}} \tag{30}$$

*D.3 Section 5.3. Proof of Theorem 4(1)*

*Proof of Theorem 4(1).*

(a) By Corollary 5(2), Proposition 7(2) and Proposition 8(2).
(b) By Corollary 5, Proposition 7(3) and Proposition 8(2).
(c) By Corollary 5 and Proposition 7(1) and Proposition 8(1).

□

## Appendix E. Calculations and proofs for Section 6

*E.1 Calculations for Section 6.1: $\alpha = \beta$*

**Proof of Theorem 4(2)** *Proof of Theorem 4(2)(a).* $X_A^{**} = X_B^{**} = \frac{\alpha}{2^{\alpha+1}}$ follows from the equations in (22) by setting $\alpha = \beta$. Setting $\alpha = \beta$ in equation (19), we obtain the following:

$$\alpha X_B^{*\alpha} = (X*_A + X*_B)^{\alpha+1}, \quad \alpha X_A^{*\alpha} = (X*_A + X*_B)^{\alpha+1} \Rightarrow X*_A = X*_B = \frac{\alpha}{2^{\alpha+1}}$$

□

*Proof of Theorem 4(2)(b).* By Corollary 6(1). □

**Proof of Corollary 5**

**Proof.**

1. From the equations in Theorem 4(2)(a), we observe that the equilibrium effort increases with $\alpha$ when $\alpha < \frac{1}{\log 2}$ and the equilibrium effort decreases with $\alpha$ when $\alpha > \frac{1}{\log 2}$:

$$\frac{\partial X*}{\partial \alpha} \geq (\leq)\frac{1-\alpha\log(2)}{2^{\alpha+1}}R \Rightarrow 1-\alpha\log(2) \geq (\leq)0 \Rightarrow \alpha \geq (\leq)\frac{1}{\log(2)}$$

Therefore, maximum effort is obtained when $\alpha = \frac{1}{\log 2}$:

$$X*_A = X*_B = X_A^{**} = X_B^{**} = \frac{1}{2elog2} \approx 0.265$$

When $\alpha \to \infty$, or $\alpha \to 0$, $X*$ and $X^{**}$ goes to 0.

2. When $\alpha = 1$, we get the EU solution: $X*_A = X*_B = X_A^{**} = X_B^{**} = 0.25$. Similarly, when $\alpha = 2$, the efforts are also 0.25. Thus, when $\alpha \in (1, 2)$, $X* = X^{**} > 0.25$, and when $\alpha \in (0, 1)$ and $\alpha \in (2, \infty)$, $X* = X^{**} < 0.25$.
3. By Theorem 4(2)(a), and by (29),

$$X*_A = X*_B = \frac{\alpha}{2^{\alpha+1}}; \quad P = \frac{X*_A}{X*_A + X*_B} = \frac{1}{2}; s^* = \frac{1}{2}. \tag{31}$$

□

**Proof of Corollary 6**

*Proof of Corollary 6(1).* The contest payoffs by (18) and Corrollary 5 are:

$$\pi_A^c(X^*{}_A)=\pi_B^c(X^*{}_B)=1-\frac{1}{2^\alpha}-\frac{\alpha}{2^{\alpha+1}}-c \tag{32}$$

$\pi_A^c(X^*{}_A)$ and $\pi_B^c(X^*{}_B)$ are greater than 0 for all $\alpha>0$. for a suitable range of $c$.
In bargaining, by restricting $\alpha=\beta$ in (22), we obtain the following:

$$s=\frac{1}{2}+\frac{1}{2^{\alpha+1}}-\frac{1}{2^{\alpha+1}}=\frac{1}{2},\quad X_A^{**}=X_B^{**}=\frac{\alpha}{2^{\alpha+1}},\quad \pi_A^b(X^{**}{}_A)=\pi_B^b(X^{**}{}_B)=\frac{1}{2}-\frac{\alpha}{2^{\alpha+1}} \tag{33}$$

$\pi_A^b(X^{**}{}_A)$ and $\pi_B^b(X^{**}{}_B)$ are greater than 0 for all $\alpha>0$.
$A$ and $B$ will prefer a contest if $\pi^{b*}<\pi^{c*}$. This requires the following:

$$\frac{1}{2}-\frac{\alpha}{2^{\alpha+1}}<1-\frac{1}{2^\alpha}-\frac{\alpha}{2^{\alpha+1}}-c\Rightarrow c<\frac{1}{2}-\frac{1}{2^\alpha}. \tag{34}$$

Notice that the *AOPC* in this setting is as follows:

$$AOPC=1-\left(\frac{1}{2}\right)^\alpha-\left(\frac{1}{2}\right)^\alpha=2\left(\frac{1}{2}-\frac{1}{2^\alpha}\right) \tag{35}$$

Therefor, the condition for contest occurring is

$$c<2APOC.$$

□

*Proof of Corollary 6(2).* $c<\frac{1}{2}-\frac{1}{2^\alpha}\Leftrightarrow\alpha\geq\frac{\log(\frac{2}{1-2c})}{\log 2},\quad c<0.5.$ □

### *E.2 Calculation for Section 6.2: $\beta=1$*

#### *E.2.1 Bargaining game efforts and outcomes, proof for Proposition 7*

From (22), we can obtain the following bargaining outcomes:

$$X_A^{**}=X_B^{**}=\frac{1}{8}+\frac{\alpha}{2^{\alpha+2}},\quad s*=\frac{3}{4}-\frac{1}{2^{\alpha+1}},\quad \pi_A^{b*}=\frac{5}{8}-\frac{\alpha+2}{2^{\alpha+2}},\quad \pi_B^{b*}=\frac{1}{8}+\frac{2-\alpha}{2^{\alpha+2}} \tag{36}$$

*Proof for Proposition 7.* We derive the comparative statics for (36):

$$\frac{\partial X_A^{**}}{\partial\alpha}=\frac{\partial X_B^{**}}{\partial\alpha}=2^{-\alpha-2}(1-\alpha\log 2)\geq(\leq)0\quad\Rightarrow\alpha\leq(\geq)\frac{1}{\log(2)}$$
$$\text{when}\quad\alpha\leq(\geq)\frac{1}{\log(2)},X_A^{**}=X_B^{**}\leq(\geq)\frac{2+e\log 2}{8e\log 2}\approx 0.2577$$

Furthermore, from (36), we can see that when $\alpha=1$ or $\alpha=2$, $X_A^{**}=X_B^{**}=0.25$. $\lim_{\alpha\to\infty}X^{**}=\lim_{\alpha\to\infty}X^{**}=0.125$ and $\lim_{\alpha\to\infty}s*=0.75$, $\lim_{\alpha\to 0}s*=0.25$, $\lim_{\alpha\to 1}s*=0.5$. Thus the range for the share of bargaining for an optimistic agent is (0.5, 0.75), and the range for the share of bargaining for a pessimistic agent is (0.25, 0.5). Accordingly the range of share from bargaining for a neutral agent is (0.25, 0.5) when faced with an optimistic opponent, and is (0.5, 0.75) when faced with a pessimitstic agent. □

#### *E.2.2 Contest game efforts and outcomes, proof for Proposition 8*

*Proof for Proposition 8.* From (19), best response contest efforts $X_A^*$ and $X_B^*$ must satisfy the following:

$$X*_A+X*_B=(R\alpha X_B^{*\alpha})^{\frac{1}{\alpha+1}}=(R\beta X_A^{*\beta})^{\frac{1}{\beta+1}}\Rightarrow X*_B=\alpha^{-\frac{1}{\alpha}}\beta^{\frac{\alpha+1}{\alpha(\beta+1)}}R^{\frac{\alpha-\beta}{\alpha(\beta+1)}}X_A^{*\frac{\beta}{\alpha}\frac{\alpha+1}{\beta+1}}$$

Set R=1 and $\beta=1$, the above conditions simplifies:

$$X*_B=\alpha^{-\frac{1}{\alpha}}X_A^{*\frac{\alpha+1}{2\alpha}},\quad X*_A+X*_B=X_A^{*\frac{1}{2}}. \tag{37}$$

Further, we infer the following:

$$X*_A+X*_B=X_A^{*\frac{1}{2}}\Rightarrow X*_B=X_A^{*\frac{1}{2}}-X*_A\quad\textit{and}\quad P=\frac{X*_A}{X*_A+X*_B}=X_A^{*\frac{1}{2}}. \tag{38}$$

By (37) and (38),

$$X_A^{*\frac{1}{2}}+\alpha^{-\frac{1}{\alpha}}X_A^{*\frac{1}{2\alpha}}=1,\quad(1-P)^\alpha=\alpha^{-1}P. \tag{39}$$

The payoffs in a contest by (18) are as follows:

$$\begin{aligned}\pi_A^c(X^*{}_A)&=1-(1-P)^\alpha-X*_A-c=1-\left(1-X^{*\frac{1}{2}}\right)^\alpha-X*_A-c\\ \pi_B^{c*}(X^*{}_B)&=1-P-X*_B-c=1-2X_A^{*\frac{1}{2}}+X*_A-c\end{aligned} \tag{40}$$

We can obtain the following comparative statics by implicit differentiation ($\alpha > 1, X*_A > 0$):

$$(39) \Rightarrow \quad \alpha^{\frac{1}{\alpha}+2} X_A^{*\frac{1}{2}} \frac{\partial X*_A}{\partial \alpha} + X_A^{*\frac{1}{2\alpha}} \left( \alpha \frac{\partial X*_A}{\partial \alpha} + X*_A (2\log\alpha - \log X*_A - 2) \right) = 0$$

$$\Rightarrow \quad \frac{\partial X*_A}{\partial \alpha} = \frac{X_A^{\frac{1}{2\alpha}+1} (-2\log\alpha + \log X_A + 2)}{\alpha \left( \alpha \alpha^{\frac{1}{\alpha}+1} X_A^{*\frac{1}{2}} + X_A^{*\frac{1}{2\alpha}} \right)} \geq (\leq) 0 \Leftrightarrow X*_A \leq (\geq) \frac{\alpha^2}{e^2}$$

$$(38) \Rightarrow \quad \frac{\partial X*_B}{\partial \alpha} = \left( \frac{1}{2} X_A^{*-\frac{1}{2}} - 1 \right) \frac{\partial X*_A}{\partial \alpha}$$

The above first order condition suggests that A's maximum best response effort is achieved when $X*_A = \frac{\alpha^2}{e^2}$. Substituting this into (39), we obtain

$$\frac{\alpha}{e} + \frac{1}{e^{\frac{1}{\alpha}}} = 1.$$

Therefore A's maximum best response effort is obtained when $\alpha^* \approx 1.3926$ (calculated with Mathematica), where $X*_A \approx 0.2625$, and by (38), $X*_B \approx 0.2498$ at $\alpha^*$. $X_A^*$ increases with $\alpha$ if $\alpha < \alpha^*$, and decreases with $\alpha$ when $\alpha > \alpha^*$.[19] Further, $X*_A = 0.25$ when $\alpha = 1$ or 2. By (39), $\lim_{\alpha\to\infty} X_A{}^* = 0$. Thus $X_A{}^* \in (0,0.2625)$. Next, notice that $\frac{1}{2}X_A^{*-\frac{1}{2}} - 1 > (<) 0$ when $0 < X*_A < 0.25 (X*_A > 0.25)$, i.e. $\alpha \in (-\infty,1) \cup (2,\infty)$ ($\alpha \in (1,2)$); and $\frac{\partial X*_A}{\partial \alpha} > 0\ (< 0)$ when $\alpha < \alpha^* (> \alpha^*)$. Thus, for party B, $\frac{\partial X*_B}{\partial \alpha} > 0$ when $\alpha \in (0,1) \cup (\alpha^*, 2)$, and $\frac{\partial X*_B}{\partial \alpha} < 0$ when $\alpha \in (1, \alpha^*) \cup (2, \infty)$. Thus $X*_B$ is maximized at $\alpha = 1$ or $\alpha = 2$, and the maximum $X*_B = 0.25$. By (38) and the fact $X*_A \in (0, 0.2625)$, we can see $X*_B \in (0, 0.25)$. Lastly, by (38) we know that $X*_A - X*_B = 2X*_A - X_A^{*\frac{1}{2}}$. This is greater than zero if and only if $X_A^* > 0.25$. In other words, $X_A^* > X_B^*$ if and only if $\alpha \in (1,2)$.□

*E.2.3 Proof of Theorem 4(3)*

Predictions on the shares of bargaining and contest-winning rates, as well as the choice between a bargaining game and a contest game, are described in Theorem 4(3).

*Proof of Theorem 4*(3)(a)

$$(38) \Rightarrow \frac{\partial P}{\partial \alpha} = X_A^{*-\frac{1}{2}} \frac{\partial X*_A}{\partial \alpha} \geq (<)\ \ 0 \text{when} \ \ \alpha \leq (>) \alpha*, \quad \alpha* \approx 1.3926$$

Thus, $A$'s winning probability in litigation $P$ increases with $\alpha$ when $\alpha < \alpha*$, and decreases with $\alpha$ when $\alpha > \alpha*$. The highest winning probability for A is when $\alpha = \alpha* \approx 1.3926$. By Proposition 8, at $\alpha*$, $X*_A \approx 0.2625$. By (38), $P \approx 0.5124$. □

*Proof of Theorem 4(3)(b).* The result for the range of bargaining share is by Proposition 7(4); By (38), A's contest winning probability is $P = X_A^{\frac{1}{2}}$. By Proposition 8(2), the approximate range of $X*_A$ is $(0, 0.2625)$. Thus the approximate range of $P$ is $(0, 0.5124)$. □

*Proof of Theorem 4*(3)(c) (33) By and (40),

$$\pi_A^{c*}(X^*{}_A) < \pi_A^{b*}(X^{**}{}_A) \quad \Rightarrow 1 - \left(1 - X_A^{*\frac{1}{2}}\right)^{\alpha} - X*_A - c < \frac{5}{8} - \frac{\alpha+2}{2^{\alpha+2}}$$

$$\pi_B^{c*}(X^*{}_B) > \pi_B^{b*}(X^{**}{}_B) \quad \Rightarrow 1 - 2X_A^{*\frac{1}{2}} + X*_A - c > \frac{1}{8} + \frac{2-\alpha}{2^{\alpha+2}}$$

$$\Rightarrow \quad 1 - \left(1 - X_A^{*\frac{1}{2}}\right)^{\alpha} - X*_A - \frac{5}{8} + \frac{\alpha+2}{2^{\alpha+2}} < c < 1 - 2X_A^{*\frac{1}{2}} + X*_A - \frac{1}{8} - \frac{2-\alpha}{2^{\alpha+2}}$$

Given a suitable cost $C$, using (38), $P = X_A^{*\frac{1}{2}}$ and using (39), $(1-P)^{\alpha} = \alpha^{-1}P$, then

$$1 - \left(1 - X_A^{*\frac{1}{2}}\right)^{\alpha} - X*_A - \frac{5}{8} + \frac{\alpha+2}{2^{\alpha+2}} < 1 - 2X_A^{*\frac{1}{2}} + X*_A - \frac{1}{8} - \frac{2-\alpha}{2^{\alpha+2}}$$

$$\Leftrightarrow 2P^2 + (1-P)^{\alpha} - 2P - \frac{1}{2^{\alpha}} + \frac{1}{2} > 0 \Leftrightarrow 2P^2 + \left(\alpha^{-1} - 2\right)P + \frac{1}{2} - \frac{1}{2^{\alpha}} > 0$$

$$\Leftrightarrow \Delta = \left(\alpha^{-1} - 2\right)^2 - 8\left(\frac{1}{2} - \frac{1}{2^{\alpha}}\right) < 0 \Leftrightarrow \alpha \gtrapprox 2.44594$$

□

*Proof of Theorem 4(3)(d).* As $0 < X_A^{*\frac{1}{2}} < 1$, when $\alpha \to \infty$, $(1 - X_A^{*\frac{1}{2}})^{\alpha} \to 0$. We also have $X*_A \to 0$ as $\alpha \to \infty$ by (36) and (40). Therefore

$$\lim_{\alpha\to\infty} \pi_A^c(X^*{}_A) - \pi_A^b(X^{**}{}_A) = \lim_{\alpha\to\infty} 1 - \left(1 - X_A^{*\frac{1}{2}}\right)^{\alpha} - X*_A - c - \frac{5}{8} + \frac{\alpha+2}{2^{\alpha+2}} = 1 - c - \frac{5}{8} = \frac{5}{8} - c.$$

[19] All the values obtained at the point $\alpha \sim 1.3926$ are approximations. For simplicity here we do not explicitly say they are approximations.

Since we have $\lim_{\alpha \to 0} X*_A = 0$, then $\lim_{\alpha \to \infty} X_A^{*\frac{1}{2}} = 0$.

$$\lim_{\alpha \to \infty} \pi_B^c(X^*{}_B) - \pi_B^b(X^{**}{}_B) = \lim_{\alpha \to \infty} 1 - 2X_A^{*\frac{1}{2}} - c - \frac{1}{8} - \frac{2-\alpha}{2^{\alpha+2}} = \frac{7}{8} - c.$$

Therefore, when $\alpha \to \infty$, for $0 \le c < \frac{5}{8}$, $\pi_A^c(X_A{}^*) > \pi_A^b(X_A{}^{**})$; and for $0 \le c < \frac{7}{8}$, $\pi_B^c(X_B)^* > \pi_B^b(X^{**}{}_B)$. □

*Proof of Theorem 4(3)(e)*. By Proposition 8(3). □

## References


Babcock, L., Loewenstein, G., 1997. Explaining bargaining impasse: the role of self-serving biases? J. Econ. Perspect. 11 (1), 109–126.
Baharad, E., Nitzan, S., 2008. Contest efforts in light of behavioural considerations? Econ. J. 118 (533), 2047–2059.
Bar-Gill, O., 2005. The evolution and persistence of optimism in litigation. J. Law Econ. Organ. 22 (2), 490–507.
Bebchuk, L.A., 1984. Litigation and settlement under imperfect information? RAND J. Econ. 15 (3), 404–415.
Binmore, K., Rubinstein, A., Wolinsky, A., 1986. The Nash bargaining solution in economic modelling? RAND J. Econ. 17 (2), 176–188.
Daughety, A.F., Reinganum, J.F., et al., 1994. Settlement negotiations with two-sided asymmetric information: model duality, information distribution, and efficiency. Int. Rev. Law Econ. 14 (3), 283–298.
Dixit, A., 1987. Strategic behavior in contests. Am. Econ. Rev. 77, 891–898.
Dummit, D.S., 1991. Solving solvable quintics? Math. Comput. 57 (195), 387–401.
Forgeard, M., Seligman, M., 2012. Seeing the glass half full: a review of the causes and consequences of optimism? Prat. Psychol. 18 (2), 107–120.
Fournier, G.M., Zuehlke, T.W., 1989. Litigation and settlement: an empirical approach? Rev. Econ. Stat. 71 (2), 189–195.
Gelbach, J.B., 2018. The reduced form of litigation models and the plaintiff's win rate? J. Law Econ. 61 (1), 125–157.
Hause, J.C., 1989. Indemnity, settlement, and litigation, or I'll be suing you. J. Legal Stud. 18 (1), 157–179.
Hirshleifer, J., Osborne, E., 2001. Truth, effort, and the legal battle. Public Choice 108 (1–2), 169–195.
Hochstadt, H., 1986. The Functions of Mathematical Physics. Dover Publications, Inc., New York, NY.
Hughes, J.W., Snyder, E.A., 1995. Litigation and settlement under the English and American rules: theory and evidence? J. Law Econ. 38 (1), 225–250.
Hylton, K.N., Lin, H., 2011. A Formalization and Extension of the Priest-Klein Hypothesis. Boston Univ. School of Law, Law and Economics Research Paper, pp. 11–28.
Jia, H., Skaperdas, S., Vaidya, S., 2013. Contest functions: theoretical foundations and issues in estimation? Int. J. Ind. Organ. 31 (3), 211–222.
Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk? Econometrica 47 (2), 363–391.
Keskin, K., 2018. Cumulative prospect theory preferences in rent-seeking contests. Math. Soc. Sci. 96, 85–91.
Kiser, R.L., Asher, M.A., McShane, B.B., 2008. Let's not make a deal: an empirical study of decision making in unsuccessful settlement negotiations? J. Empir. Legal Stud. 5 (3), 551–591.
Konrad, K.A., 2009. Strategy and Dynamics in Contests. Oxford University Press.
Konrad, K.A., Morath, F., 2017. Escalation in Dynamic Conflict: On Beliefs and Selection.
Korobkin, R., 2005. Psychological impediments to mediation success: theory and practice. Ohio State J. Disput. Resolut. 21, 281.
Korobkin, R., Guthrie, C., 1997. Psychology, economics, and settlement: a new look at the role of the lawyer. Texas Law Rev. 76, 77.
Lee, Y.-H.A., Klerman, D., 2016. The Priest-Klein hypotheses: proofs and generality. Int. Rev. Law Econ. 48, 59–76.
McBride, M., Skaperdas, S., Tsai, P.-H., 2018. Why go to court? Bargaining failure under the shadow of trial with complete information. Eur. J. Polit. Econ. 55, 151–168.
Munsinger, H.L., Philbin Jr., D.R., 2016. Why can't they settle: the psychology of relational disputes. Cardozo J. Confl. Resolut. 18, 311.
Nash, J., 1950. The bargaining problem? Econom.: J. Econom. Soc. 18 (2), 155–162.
Nash, J., 1953. Two-person cooperative games? Econom.: J. Econom. Soc. 21 (1), 128–140.
O'Grady, C.G., 2006. Cognitive optimism and professional pessimism in the large-firm practice of law: the optimistic associate. Law Psychol. Rev. 30, 23.
Polinsky, A.M., Rubinfeld, D.L., 2002. A note on settlements under the contingent fee method of compensating lawyers? Int. Rev. Law Econ. 22 (2), 217–225.
Priest, G.L., Klein, B., 1984. The selection of disputes for litigation? J. Legal Stud. 13 (1), 1–55.
Quiggin, J., 1982. A theory of anticipated utility? J. Econ. Behav. Organ. 3 (4), 323–343.
Quiggin, J., 1991. On the optimal design of lotteries? Economica 58 (229), 1–16.
Quiggin, J., 2012. Generalized Expected Utility Theory: The Rank-Dependent Model. Springer Science & Business Media.
Robson, A., Skaperdas, S., 2008. Costly enforcement of property rights and the coase theorem? Econ. Theory 36 (1), 109–128.
Rubinstein, A., 1982. Perfect equilibrium in a bargaining model. Econom.: J. Econom. Soc. 50, 97–109.
Shavell, S., 1982. Suit, settlement, and trial: a theoretical analysis under alternative methods for the allocation of legal costs. J. Legal Stud. 11 (1), 55–81.
Shavell, S., 1996. Any frequency of plaintiff victory at trial is possible? J. Legal Stud. 25 (2), 493–501.
Skaperdas, S., Gan, L., 1995. Risk aversion in contests? Econ. J. 105 (431), 951–962.
Tversky, A., Kahneman, D., 1992. Advances in prospect theory: cumulative representation of uncertainty? J. Risk Uncertain. 5 (4), 297–323.
Waldfogel, J., 1995. The selection hypothesis and the relationship between trial and plaintiff victory? J. Polit. Econ. 103 (2), 229–260.
Yaari, M.E., 1987. The dual theory of choice under risk? Econom.: J. Econom. Soc. 55 (1), 95–115.